Patterns for turning

Sir Howard Warburton Elphinstone

PATTERNS FOR TURNING.

Plate 62.

PATTERNS FOR TURNING:

COMPRISING

Elliptical and other Figures Cut on the Lathe

WITHOUT THE USE OF ANY ORNAMENTAL CHUCK.

By H. W. ELPHINSTONE.

WITH SEVENTY ILLUSTRATIONS.

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PREFACE.

ALL the Patterns contained in this book can be cut on a lathe furnished with a division-plate, an ornamental slide-rest, an eccentric cutting-frame, and an overhead motion. The reader is requested to peruse this book by the side of his lathe, and to perform the various operations indicated in the text; he will then, it is hoped, find no serious difficulty in understanding the methods of cutting the Patterns.

Most of the Patterns can be cut by a person capable of performing simple addition or subtraction of numbers containing two figures, and the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. Some few require a knowledge of decimal fractions.

Brief explanations have been inserted of the geometry of the Patterns, the paragraphs containing such explanations are introduced by the words "the mathematician will observe," or the like, for the sake of warning the non-mathematical reader to avoid them.

The author particularly directs the attention of those among his readers who have an elementary knowledge of mathematics to the method of "Envelopes." He hopes that any one who invents any fresh patterns by this method will communicate them to him.

APRIL, 1872.

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PATTERNS FOR TURNING.

CHAPTER I.

GENERAL EXPLANATIONS.

1. THROUGHOUT this book measurements of length are expressed in hundredths of an inch, or in other words ·01 ($\frac{1}{100}$) of an inch is taken as the unit of length: it follows that one inch will be expressed by 100. It is convenient to express measurements in this manner, for in a modern lathe made by any one of the best makers, the screw both of the slide-rest and of the eccentric cutting-frame contains ten threads to an inch, and the screw thread is graduated into ten equal divisions: it follows that when the screw-head is moved through one division the slide either of the slide-rest or of the eccentric cutting-frame will be made to traverse through ·01 ($\frac{1}{100}$) of an inch, or as we shall call it, through 1.

2. Unless the contrary is expressly stated, the motion

of the slide both of the slide-rest and of the eccentric cutting-frame is always to be made forwards, *i.e.*, so as to move the tool-holder of the slide-rest towards the left, and to move the tool-holder of the eccentric cutting-frame, so that a larger circle can be described with that instrument after the tool-holder has been moved, than could be described before this was done. It follows that the direction "move the tool-holder of the slide-rest through 98," means, "turn the screw forward through nine whole turns and eight divisions." Similarly, the direction, "throw out the eccentric cutting-frame 81," means, "move the screw forward through eight turns and one division." On each of these screws there are divisions indicated by short lines midway between the numbered divisions; a motion of the screw from one of the divisions indicated by a short line to one of those marked by a number (or *vice versâ*) moves the slide through ·005($\frac{5}{1000}$) or $\frac{1}{2}$ a hundredth of an inch, or according to our notation through ·5 or $\frac{1}{2}$.

3. In most patterns an error of $\cdot\frac{25}{2}$ (or $\frac{1}{8}$ of a hundredth of an inch) may be neglected, and accordingly the author has, as a general rule, given the settings to the nearest ·0025 or $\frac{1}{4}$ of a hundredth of an inch, or according to

our notation to the nearest $\frac{1}{4}$. In some few patterns, and in the patterns with a variable radius (Chapter V.) a still higher degree of accuracy should be aimed at. With a little practice the workman will be able to subdivide by eye the distance between a long and a short division on the screw-head of the slide-rest screw into five equal parts, corresponding to ·1 according to our notation (*i.e.*, to ·001 or $\frac{1}{1000}$ of an inch); unfortunately, however, this cannot be done on the screw-head of the eccentric cutting-frame, as the diameter of the screw-head is smaller than that of the screw-head of the slide-rest, and the divisions are more coarsely marked than those of the latter screw-head. This signifies the less that, as will be explained (*post*, Article 10), the play of the tool in the tool-holder of the eccentric cutting-frame may introduce a considerable error. It follows that in patterns where great exactness is required, the exact distance that the eccentric cutting-frame ought to be thrown out can only be determined by trial. If, as usually happens, the pattern has to be cut deeply, the trial may in most cases be made on the work itself by making a mere scratch with the tool set in what is believed to be the proper place, and if the circle described should not be of the size required, the tool-holder can be moved a little backwards or forwards, until the circle described is of the proper size. In some few cases it will be found convenient to make the trials on a plain box-wood chuck before the work is placed in its position.

4. *Definition.*—When the longer slide of the ornamental slide-rest is parallel to the lathe bearers, the slide-rest is said to be in its *direct* position. When the longer slide is at right angles to the lathe bearers, the slide-rest is said to be in its *transverse* position; and when the longer slide is in any intermediate position between those mentioned, the slide-rest is said to be in an *oblique* position.

5. *Definition.*—Let the tool-holder of the slide-rest be in such a position that the centre of a circle cut with the eccentric cutting-frame would coincide with the centre of the work; the distance that it has to be moved forwards, so as to enable a given circle to be cut, is called the eccentricity of that circle.

When the slide-rest is in its direct position, this definition loses its meaning; and by the eccentricity in this case is meant only the distance that the tool-holder has to be moved forwards from some determinate position.

When the slide-rest is in its transverse position, the eccentricity of any circle, as above defined, equals the distance of its centre from the centre or from the axis of the work; when the slide-rest is in any other position, the eccentricity of any circle is greater than the distance of its centre from the axis of the work: this latter distance is called the "true eccentricity." The mathematician will observe that when the slide-rest is oblique, we have—

True eccentricity = eccentricity × sine of the angle of obliquity.

6. If the tool-holder of the slide-rest be moved in one direction by turning the screw of the slide-rest, and the screw be afterwards turned in the opposite direction, it will be found that it can be turned through a small distance (one or two divisions of the screw-head), without producing any corresponding motion of the tool-slide; this fact, and sometimes the number of divisions through which the screw-head can be turned without producing any motion of the tool-slide, is called "loss of time." It is occasioned by the nut not fitting the screw exactly. When we move anything towards the

left by pushing it, we push against its right side, and *vice versâ:* so when the screw pushes the nut of the tool-holder towards the left, the thread of the screw bears against the right-hand side of the thread of the nut. Now let the screw be moved in the opposite direction; if the screw and nut fitted exactly, the thread of the screw would immediately come into bearing on the left-hand side of the thread of the nut; but as this is not the case, the screw will be moved through a small distance before its thread pushes against the left-hand side of the thread of the nut—*i. e.*, before the nut begins to move towards the right. A little reflection will show that loss of time must always exist wherever a screw works in a nut, for taking the extreme supposition that a new screw and nut could fit so exactly that the loss of time was inappreciable, the wear produced by use would soon give rise to it. The existence of loss of time must be carefully remembered when the workman attempts to cut any pattern which requires great accuracy.

Let us suppose the slide-rest to be in the transverse position, the amount of loss of time to be two divisions, and that we wish to cut a circle whose eccentricity is

100. Adjust the tool-slide of the slide-rest, so that a circle cut with the eccentric cutting-frame has no eccentricity, or in other words, so that its centre coincides with the centre of the work; then if it has been brought to this position by a movement from the right, no loss of time will occur in moving the tool-slide forward through 100: but if the tool-slide has been put into this position by moving it from the left, loss of time will occur when we move it towards the left, and when we have turned the screw-head through 100 divisions, the slide will have only moved through $100 - 2 = 98$. Suppose, however, that we move it towards the left by turning the screw through 105 instead of 100 divisions, the tool-slide has been moved through 103 only; now move the screw in the contrary direction through five divisions, the two first of these will produce no movement of the slide-rest, the remaining three will place it in the position required, namely, 100 from the central position.

It appears therefore that there are two methods of placing the tool-holder into position for cutting a circle of given eccentricity; *first*, place the tool-slide to the right of the centre by a distance greater than the loss of time, then move it forward till a circle cut with the eccentric cutting-frame would have no eccentricity, or would have its centre coincident with the centre of the work; note the reading, and turn the screw forward through the number of divisions corresponding to the given eccentricity: *secondly,* place the tool-holder to the left of the slide-rest, by a distance greater than the loss of time, turn it backwards till a circle cut with the eccentric cutting-frame would have no eccentricity, or would have its centre coincident with the centre of the work; then turn it forwards through a number of divisions greater by (say) five than the given eccentricity; and lastly turn it back through five divisions.

Although we can place the tool-holder in its proper position for cutting the required circle by either of these methods, and therefore, if the pattern consists entirely of circles of the same eccentricity, we can adopt either method indifferently, this is not the case if circles of different eccentricities have to be cut in a subsequent part of the pattern: for if the first method is adopted and the second cut has to be made with less eccentricity than the first cut, loss of time will occur in moving the tool-holder into position for it; similarly if the second method be adopted, loss of time will occur in moving the tool-holder into position for a cut with greater eccentricity. It follows that in cutting a pattern we must always begin with the cut that has the least or with that which has the greatest eccentricity; if we begin with that which has least eccentricity we adopt the first method for putting the tool-holder into position for the first cut, while if we begin with that which has the greatest eccentricity we adopt the second method. The author has found it most convenient in practice to begin as a *general* rule with the cut farthest from the centre, and therefore to adopt the second plan; he only begins with the cut of least eccentricity when that eccentricity is zero.

The reader may think the foregoing discussion frivolous, and that in practice loss of time may be disregarded. Although in many patterns this is the case, others would be entirely ruined by allowing the loss of time to affect the position of the cut. Most of the envelopes afford examples of this (see the caution given, p. 127). In cases where the workman desires to place circles exactly in contact (*post*, p. 34) he must be most careful to avoid loss of time; the difference in the position of the circles produced by neglecting it is sufficient to prevent them from being in contact when the workman thinks that he has employed the values of the eccentricity and radius calculated by the tables. (See Plates 4 and 54.)

The following is a curious example of the effects of loss of time. The figures printed in this book were all cut with a tool placed in the eccentric cutting-frame; in cutting the figures it was necessary to make the cuts extremely shallow, in fact to make little more than scratches. I adopted the usual plan of letting the tool very gently into cut by means of the shorter regulating screw of the tool-holder of the slide-rest; when the cut appeared to be of the proper depth, I withdrew the cutting-frame by the longer regulating screw of the tool-holder, I then clamped the shorter screw and proceeded to make the subsequent cuts in the usual manner. I very often found that the depth of the subsequent cuts was rather less than that of the first cut, although according to the rules usually laid down, the depth of every cut should have been the same. The following is the explanation of the phenomenon. The screw in question works in a nut split along its length; the operation of clamping is produced by tightening the two halves of the nut on the screw. When the tool is brought into cut while the nut is unclamped, the screw has a considerable amount of play in the nut, and as the tool-slide of the slide-rest is pushed forwards by the lever, the thread of the screw touches the front side of the thread of the nut, and as the two parts of the nut do not quite touch each other, the edge of the thread of the screw projects backwards through the interstice; now let the nut be clamped, the two parts of it are pushed towards each other until they meet, and as they come in contact they push the screw thread from between them—in other words they push it forwards; it follows that when the tool is let up into cut after the nut has been clamped it will not cut quite so deeply as before. The remedy is obvious: before you make the first cut turn the clamping screw so that the shorter regulating screw will only move stiffly, and after you have clamped it keep the tool in the position for the first cut, and endeavour to let it up into cut by the movement of the longer regulating screw; should it not quite come up into cut—an event which will rarely happen if the above-mentioned precaution of making the shorter regulating screw move stiffly be taken—that screw must be unclamped and the screw turned backwards through a very small distance, it must then be clamped again and a fresh trial made : in making the trial I find that the ear can detect whether the tool just comes into cut, with greater accuracy than the eye can.

7. *Definition.*—The reading of the screw-head of the traversing screw of the slide-rest, when the slide-rest is either in the transverse or an oblique position, and when the tool-holder is in such a position that a circle cut with the eccentric cutting-frame would have no eccentricity, or in other words when the centre of such a circle would coincide with the centre of the work, and when the tool-holder has been brought into that position according to such one of the rules above given for avoiding loss of time as ought to be adopted, having regard to the nature of the pattern, is called the *central* reading, and the slide-rest is said to be in its central position.

If the slide-rest is in its direct position the reading of the screw-head, at the position whence the eccentricities are measured, is, when it has been brought into that position according to such one of the rules above given for avoiding loss of time as ought to be adopted, having regard to the nature of the pattern, called the *central* reading.

In order to place the slide-rest accurately in the central position, two adjustments are necessary: first, the slide-rest must be adjusted for height, an adjustment which is effected by means of the elevating screw; secondly, the tool-holder must be moved laterally by means of the traversing screw of the slide-rest either to the right or left, according to the rules above given for avoiding loss of time, until it is found upon trial that the tool is in the required position.

8. In a modern slide-rest, made by one of the best makers, it will generally be found that when the height of the slide-rest has been adjusted for one of the slide-rest tools, the adjustment is correct for a tool in the eccentric cutting-frame; but this is not always the case, and it may become necessary, when the cutting-frame is substituted for the slide-rest tool, to make the adjustment afresh. In cases where during the process of surfacing the work a small knob has been left projecting in the centre of the work, this can be done by trial; but sometimes it is impossible to leave the knob, and in this case the easiest plan is to employ a brass standard, originally suggested by the late Captain Ash in his treatise on Double Counting, but now made according to an improved pattern by Messrs. Holtzapffel,

standing on the lathe bearers, and marked at the height of centre. In the absence of such a standard, the easiest plan to follow in cases where the knob has not been left, is to take the work off the mandrel, without disturbing it in the chuck, and to substitute for it a plain box-wood chuck. This can be turned up so as to leave a projecting knob by means of which the adjustment can be made, and afterwards the work can be replaced. Occasionally it is impossible to disturb the work: in such cases the author adopts the following plan. He adjusts the tool with which the pattern is to be cut in such a position in the eccentric cutting-frame that it would cut a mere dot. He then places the eccentric cutting-frame in the tool-holder of the slide-rest in a reversed position, so that the pulley is next the work; the tool will then point away from the work, and, on the back poppet-head with a pointed centre in it being placed close to the tool, the adjustment for height can readily be made. There is an obvious objection to making the adjustment in this manner; viz. that if the slide-rest be tilted in the slightest degree the adjustment will be false. If the slide-rest is properly constructed, and has met with fair usage, dirt is the only cause of tilting. The workman should therefore, before he puts the slide-rest on the lathe, most carefully remove all chips from the lathe bearers; he should also occasionally remove the body of the slide-rest from the poppet in which it stands and remove the chips which have accumulated between the body and the poppet. If he takes these precautions there will be but little risk of any error arising from the slide-rest being tilted.

9. *Definition.*—The apparent eccentricity of any cut is the reading of the traversing screw of the slide-rest at the moment when it is made.

In every position of the slide-rest we have—

$$\text{Apparent eccentricity} = \text{eccentricity} + \text{central reading};$$

when the slide-rest is in its transverse position we have—

$$\text{Apparent eccentricity} = \text{true eccentricity} + \text{central reading}.$$

10. *Definition.*—When a tool in the eccentric cutting-frame is adjusted, having regard to the rules for avoiding loss of time, in such a position that it will cut a mere dot, the tool and sometimes the cutting-frame itself is said to be in the central position.

It must be remembered that when the tool is in the central position, it does not necessarily happen that the reading of the index of the screw-head of the eccentric cutting-frame is zero, a fact which may occur for three reasons: *first*, owing to loss of time; *secondly*, there may be an index error in the screw-head, or in other words, the graduation of the screw-head may not begin in the right place. Although this index error is but small in a well-made cutting-frame, and its maximum value is half a turn of the screw, its existence must be remembered. Probably the amount of the error varies from time to time, and as in practice the central position of the tool is determined by trial, the numerical value of this error is not material. *Thirdly*, it is impossible to make the tool fit exactly into the tool-holder, it must be a little smaller so as to allow of its being put in and of its being taken out readily; and even if it were to be made to fit tightly it would soon become loose by wear. The last-mentioned error is one of considerable magnitude; a full discussion of it will be found in Engleheart's "Eccentric Turning," p. 53.

In practice the author proceeds in the following method for the purpose of avoiding the errors mentioned in this article. He places a plain box-wood chuck on the lathe, he brings it to a fine surface, using the slide-rest in doing so; he then places the eccentric cutting-frame, holding the tool with which the work is to be cut, into the slide-rest; he puts the tool, by the movement of the traversing screw of the eccentric cutting-frame, into such a position that it will cut a mere dot, making such movement to the right or left as may be necessary according to the rules above laid down (Article 6, p. 8) for avoiding loss of time, keeping his thumb during the whole process firmly pressed against the tool towards the right. He then notes the reading of the screw-head of the eccentric cutting-frame, which may be called the central reading of the eccentric cutting-frame, and throws out the tool-slide the number of divisions required for the first cut, keeping his thumb firmly pressed against the tool towards the right during the movement. The following method would be still more accurate: Throw out the slide of the eccentric cutting-frame to what is believed to be the required radius, just scratch a circle, move the tool-holder of the slide-rest (avoiding loss of time) through a distance equal to twice the radius, scratch another circle; the two circles will exactly meet if the amount by which the slide of the eccentric cutting-frame has been thrown out is correct, if not a slight alteration must be made, and the process must be repeated till the circles come exactly in contact.

The reader may inquire whether such accuracy is necessary in practice. Perhaps the best answer that can be given is to request him to compare the figures in Engleheart's "Eccentric Turning" with those in the "Handbook of Turning." The greater beauty of those in the first-mentioned book is incontestable; very careful examination will show that part at least of the difference arises from the fact that in the patterns given in the one book circles that ought to touch are made to touch exactly, while in the other this is not the case.

The reader will probably have gathered that in all cases where the workman cuts two circles with such radius and eccentricity that they ought to be in contact, and it becomes necessary, owing to a slight error, to alter either the eccentricity or the radius by a small quantity so as to make the contact exact, it is most likely that the error exists in the radius.

11. *Definition.*—By the direction "all at centre" is meant that the eccentric cutting-frame is to be placed in

its central position and that the slide-rest is to be placed in its central position. In other words the tool is in such a position that, whether the tool revolves while the mandrel remains fixed, or the mandrel revolves while the tool remains still, a mere dot will be described.

12. Throughout this work the letters *r*, *e*, *n*, stand for radius, true eccentricity, and number of cuts respectively; and unless the contrary is directed, it is supposed that the cuts are disposed equally round the work by means of the division-plate, and that the slide-rest is in the transverse position. Thus the direction

$$e = 40 \qquad r = 20 \qquad n = 8$$

means, throw out the tool-holder of the slide-rest four turns of the screw from the position all at centre, throw out the slide of the eccentric cutting-frame so that the tool in it will cut a circle whose radius equals 20, *i.e.*, throw it out about 20 divisions from its central position, make cuts at the numbers

96, 12, 24, 36, 48, 60, 72, 84

on the division-plate, scale 96.

It is obvious that instead of cutting at these numbers we might have begun at any number, 7 for instance, and made cuts at every twelfth number, that is at

7, 19, 31, 43, 55, 67, 79, 91

on the scale of 96. It is hardly necessary to add that the scale of 120 might have been chosen; we might have cut at

15, 30, 45, 60, 75, 90, 105, 120.

The direction "cut at any particular numbers," means, make cuts with the given radius and eccentricity at these numbers on the division-plate, scale of 96, unless some other scale is expressly mentioned.

13. The settings of all the patterns given in this book are given on the supposition that the slide-rest is in the transverse position. Some of them can be cut without much change of character when the slide-rest is in a slightly inclined position; and in this case, if no cuts of the pattern are required to meet exactly, we may take the given value of *e* (the true eccentricity) as the eccentricity. If, however, some of the cuts are required to meet, it must be remembered that the true eccentricity must be used.

Some few of the patterns look very well cut with the slide-rest in the direct position, using a round tool in the horizontal cutting-frame of such a width that the cuts exactly meet, making flutes of a length which equals the eccentricity given in the setting for the pattern.

14. The beauty of the patterns depends in great part on the facets of the cuts being brilliantly polished, and on the cuts themselves being all of the same depth. It must not be imagined that the pattern when cut will present the same appearance as it does when printed. It is hardly possible to distinguish the outline of some of the most brilliant patterns in certain lights, as the reflection from the facets catches the eye and draws the attention off the outline; and as the position of the eye is changed, the appearance of the reflection is changed also, thus producing a play of light. A familiar example of the difference between the outline of the pattern and the form of the figure produced by the play of light from the facets, is the "figure of eight" produced by the reflection from a "Turk's head." As a general rule, in selecting a pattern for ornamenting a piece of choice work, the workman should turn his attention towards the selection of a pattern which produces a fine play of light, rather than one which has a fine outline.

There is no difficulty in producing a fine polish on the facets of the cuts; the workman must polish his tool very carefully before he begins his work, and must let it up into cut very gradually by means of the regulating screw. If the tool has to do a great deal of work before the pattern is complete, it is desirable, if possible, not to make the cuts to the full depth in the first instance, and after all the cuts have been made to re-sharpen the tool, and to go over the pattern again, making the cuts of the full depth. It must be remembered, however, that the difficulty of placing the tool in exactly the same position in the eccentric cutting-frame is very great, and that therefore this course cannot be followed where the cuts are fine.

It appears at first sight that there can be no difficulty in making all the cuts to the same depth, so long as the tool is firmly clamped in the tool-holder of the eccentric cutting-frame by the binding screw, and the shorter regulating screw of the slide-rest tool-holder has been firmly clamped with the precautions stated *ante,* Art. 6, after the first cut has been made. Care must, however, be taken that the slide-rest itself is very firmly fixed in its position by the screw that holds it down on the lathe-bearers, and that the longer slide is very firmly fixed in its position by means of the clamping screw, or it may happen that either the whole slide-rest or its longer slide may make a very slight movement during the process of cutting the work, which will thus be spoilt. Care must also be taken that no dirt or chip is between the shorter regulating screw and the pillar against which it abuts, and for this reason it is advisable to keep the slide-rest and the neighbouring part of the lathe scrupulously clean. The author always keeps a common brush within reach, with which he sweeps away the shavings as they accumulate from time to time.

In a pattern cut with a variable radius—in the common shell for example—there is another source of error as to the depth of cut. Unless the tool is pushed quite home into the tool-holder, there is a risk that when in the process of cutting the pattern the radius is altered from time to time, the tool may be pushed a little farther into or pulled a little further out from the tool-holder; in patterns of this nature, the eccentric cutting-frame must be carefully kept free from dirt and chips, or on a movement of the tool-holder they may get behind the tool, thus pushing it a little nearer to the work, and rendering the cuts deeper. The author has seen an eccentric cutting-frame covered with rust; it appears needless to add that the workman who used it was unable to produce any accurate work with it.

15. It is of great importance to throw a good light on the work, though it is sometimes difficult to do so, particularly if the workman labours under the same disadvantage as the author, viz. that of being unable to work during the day time, and being, therefore, dependent on artificial light. For this purpose the author adopts one or other of two plans, according to the position of the work, for the purpose of concentrating the light on it. He sometimes uses a common condensing lens, mounted on a stand; at other times, when the work lies between him and the source of light, he uses a small mirror (an old microscope mirror) fixed into a piece of wood, cut so as to fit on the screw-head which holds down the tool in the tool-holder of the slide-rest.

16. All the figures given in this book have been cut by the author; the greater number of them have been cut on boxwood or cocus wood, and some few have been cut on type metal. The author desires to acknowledge the invaluable advice and assistance given to him by his

friend Mr. Macrory towards the preparation of the blocks for printing; it is not too much to say that without them the author would have been unable to cut any blocks worth printing from.

As some of his readers may wish to cut patterns for printing, he here states the plan that he has adopted. The workman should prepare a number of boxwood, or preferably cocus wood, disks of the same diameter; the thickness of each disk may either be equal to the height of type, *i.e.* $\frac{19}{20}$ of an inch, in which case the block when cut may be printed from without any further preparation, or it may be about $\frac{4}{10}$ of an inch; it is more convenient for the workman, for some reasons, to adopt the latter plan, but if it be adopted, it becomes necessary to mount the disks before printing from them, so as to bring them to the proper height. A spring chuck must also be made that will just take the disks.

Chuck a disk, set the slide-rest as accurately as possible in the transverse position, put a very sharp roughing tool into the tool-holder, with one deep cut make the surface of the work level, leaving a minute knob projecting on the middle of the block, examine the surface of the work with a straight-edge, and if necessary readjust the slide-rest more accurately into the transverse position; adjust also for height of centre. Unless the workman possesses the brass standard mentioned above (Art. 8), he should now place the eccentric cutting-frame, containing the tool with which the work is to be cut, in its central position into the tool-holder of the slide-rest, and observe and write down for future use the central reading of the slide-rest; finally he must replace the roughing tool in the slide-rest and turn off the minute point. The face of the work that has been operated on should be carefully examined, and if it be free from deep flaws or cracks it may be prepared in manner following for having the pattern cut on it. Re-sharpen the roughing tool, inspect the surface of the work to see if there be any minute flaws in it, if so the next cut must be made deep enough to cut them out; if necessary this process must be repeated again and again until no flaws are visible. In the author's experience it is important to make the successive cuts deeper than any flaw that is visible, so as to cut it out, and not to cut through it. As soon as the wood appears free from flaws re-sharpen your tool very carefully, shift the gut to the quickest speed, and make a cut which reduces the work by the smallest possible quantity, say

·25 or ·0025 of an inch, running the lathe very quickly indeed; make two or three more cuts, each of about ·25 or ·0025 of an inch in depth; the surface should now appear brilliant, if it does not the tool must be very carefully re-sharpened and the process repeated. Lastly, turn off the circumference to about the depth 10, so as to avoid the risk of a ragged edge. The author was not aware of the importance of doing so until he had cut most of the blocks from which the figures given in this book are printed, and accordingly the ragged edge will be seen in many of them. All the cuts should be made by cutting from the circumference towards the centre of the disk, for when they are made in the contrary direction the wood is apt to become porous. It has been suggested by an amateur of great experience that it might be necessary to make the cuts in the contrary direction on some pieces of wood; but the author has never found this to be the case. Should any deep flaw or knot appear in the wood during the above process it may be cut out and the process begun *de novo*, or, as is generally more convenient, the disk may be taken out of the chuck and re-chucked so as to present the other side to the workman, and the same process must be repeated on that side.

If the pattern were now to be cut on the block it is probable that, unless the wood were of exceptionably fine quality, the prints would present a mottled appearance; most of the prints in the "Handbook of Turning" are disfigured by this. The author has not been able entirely to overcome this difficulty, as will be seen by looking at the figures, but after many experiments he has adopted the plan of lacquering the wood when in this state. It does not appear to be necessary to polish the lacquer, it is only necessary to fill the pores of the wood with it. It is desirable before the application of the lacquer to apply a piece of flannel to the wood while the lathe runs fast, this appears to give a higher degree of polish to the surface. The author employs a few drops of Messrs. Holtzapffel's "Hardwood lacquer" on a piece of cotton wool the size of a walnut, covered with old linen rag on which a drop of linseed oil is placed.

He has found that if the rubber charged with lacquer be put away after use in an air-tight box (he uses one of the smallest of the tin canisters in which tobacco is sold,) it can be used over again without being replenished with lacquer.

It is of the utmost importance not to remove the slide-rest when the lacquer is applied, on account of the difficulty of replacing it exactly in its original position ; but it will require a little practice before the workman will be able to apply the lacquer without doing so.

There is considerable difficulty in obtaining sound castings of type metal: they are apt to contain air-bubbles, and to be full of sand. The best plan is to purchase some of the blocks prepared for tinting ; these can be obtained cut to any shape required, and are reasonably sound. In preparing a block of type metal to receive a pattern, the author follows nearly the same method as that given above for the preparation of a block of wood. The following points, however, must be attended to. *First.* The lathe should be run at the slowest speed given by the large driving wheel: except toward the end of the process, when the quickest speed should be used. *Secondly.* The tool is apt to chatter, this can be prevented by placing a small piece of flannel, folded twice, between the tool and the piece of brass by which it is held down in the tool-holder. *Thirdly.* When a shaving is cut off, its under surface is chemically clean, as is also the surface from which it has been cut; and, as there is a strong tendency for two pieces of type metal having chemically clean surfaces to adhere to each other, it is found that towards the end of the operation, when the thickness of the metal removed is inconsiderable, the small pieces which are detached adhere to the finished surface, thus producing a roughness. This may be completely prevented by smearing the surface of the block with a little oil before the finishing cuts are taken. *Fourthly.* As there is no grain in the metal the cuts may be made at pleasure, either from the circumference towards the centre or *vice versâ.* For the finishing cuts the author prefers to run the tool very slowly backwards and forwards, without making any change in the depth of cut. *Fifthly.* The work should, after the finishing cut has been made, be removed from the lathe without disturbing it in the chuck. An examination of it under a good light will probably show many scratches. It is difficult to discover the reason of their occurrence; the author is inclined to think that they are produced by very small particles of the metal, which after being cut off get jammed between the tool and the face of the work. The scratches can be got rid of by rubbing the work, as it revolves in the lathe, with a little crocus powder put on flannel with oil.

The block is now prepared for having the pattern cut upon it. In order to do so the tool with which the pattern is to be cut, which should be a double-angled tool of about 50, carefully sharpened and polished on the goneometer, must be carefully adjusted in its central position in the eccentric cutting-frame. Place the eccentric cutting-frame into the slide-rest, adjusted, as before-mentioned, for height of centre, and in those many cases where it is necessary to do so, place it in the position of all at centre (see *ante*, p. 18). The workman will now be ready to set the eccentricity and radius, and the division-plate index for the first cut. It is unnecessary to give any further explanations with respect to cutting the pattern, except to say that it must be merely scratched.

The prudent workman will write down on a piece of paper the adjustments for each cut, keep the paper on the lathe before him and mark off each cut as soon as it is made; for many a fine piece of work has been destroyed owing to the workman's attention having been called off for a moment, and his consequently having forgotten what cut ought to be made next. Lastly, if the pattern has been cut on the side of the block first operated on, the work must be reversed in the chuck and the back turned off level.

When the pattern is to be cut on type metal, the author prefers to run the overhead motion from the small driving wheel of the lathe. The effect of a pattern cut on type metal before it becomes tarnished, examined by the light of a single gas jet, is extremely brilliant.

17. If the block be not intended for printing from, most of the directions given in the last article should be followed, but the slide-rest is not necessarily placed in the transverse position, and the surface of the work must either be covered with lacquer or French polish brought to a brilliant surface (see a paper by the Rev. S. Burnaby in the "Quarterly Journal of the Amateur Mechanical Society," Vol. I., p. 93) or, as the author prefers, should be grailed.

It is hardly necessary to remind the workman that the operation of grailing must be performed most carefully, with a highly polished tool. The facets of wood left between each cut should be brought to an edge in front, when viewed through a lens. Some of the patterns contained in this book which require very fine cutting look best when the grailing is made very coarse.

As a general rule the patterns look best when cut with a double-angled tool, though some few, as "Basket-work" for example, look best when cut with a left-side single-angled tool. There is a controversy between workmen as to whether the patterns cut with a double-angled tool produce the best effect when the angle of the tool that is ground away is small, say 18° or 22°, or when it is large, say 35°. In the author's opinion tools of large or small angles should be used for different purposes; if the beauty of the pattern depends upon the play of light (Article 14), he employs a tool of 18° or 22°, for in that case a small change in the position of the eye from the position in which it sees the light reflected from one side of the facets enables it to see the light reflected from the other side of the facets, in other words the play of light is readily produced; on the other hand, if the beauty of the pattern consists in the form, he employs a tool of 35°, for in that case the eye must undergo a considerable change in position in order to produce the play of light.

18. Should the workman be so unfortunate as to make a false cut, he must not lose his presence of mind. If the block is intended to be printed from it is spoilt, but if this is not the case, a slight alteration can generally be made in the pattern so as to include the false cut among those that ought to have been made. An example of the effect of a false cut in spoiling a pattern and of the method of making a slight alteration, so as to include the false cut, will be found at Figs. 37 and 61 (see the description of these figures).

CHAPTER II.

ON PLACING CIRCLES IN CONTACT.

1. It often happens in designing a pattern which is intended to contain circles cut at numbers equidistant from each other, on any scale of the division-plate, with the same radius and eccentricity, but having the radius less than the eccentricity, that the circles may be made according to the taste of the workman, either to touch each other exactly or to overlap a little; it will generally be found that the beauty of the pattern is much enhanced by causing the circles to touch each other exactly, and that, if they do not do so, they can be caused to do so without much alteration in the general design of the pattern, by a slight change of one of the quantities, the radius, the eccentricity, and the number passed over on the division-plate between any two consecutive cuts.

Although in practice, before the workman cuts a pattern, he ought, for the reasons stated, ante, p. 17, to test the correctness of the settings of any two circles that are intended to be in contact, by merely marking them with the point of the tool, it is extremely convenient to be able to determine the values of the radius and eccentricity before beginning the work. For this reason, the author subjoins rules for doing so, originally published by him in the Quarterly Journal of the Amateur Mechanical Society; the rules will be found to comprise rules for basket-work. (See for explanation of the term basket-work, Chapter III. Art. 3.)

2. RULES FOR DETERMINING THE SIZE, NUMBER, AND PORTION OF CIRCLES IN CONTACT.

GIVEN, any one of the three quantities, the radius, the true eccentricity, and the number of circles in contact, to determine the other two. This can readily be done by "TABLE FOR CONTACT OF CIRCLES, No. 1," post, p. 185.

This table consists of two columns, the one headed "Number of Circles," the other headed, "Modulus."

By the modulus corresponding to any given number of circles, is meant the number in the column headed "Modulus," opposite to the given number, and, conversely the number of circles corresponding to any given modulus is the number in the first column corresponding to the given modulus in the second.

In the following rules for shortness I write "number," or "n," for the number of circles, and "m" for modulus, "r" for radius, "e" for true eccentricity.

RULE 1.—Given eccentricity and number, to find radius.

Radius = eccentricity × modulus.

Example.—Let $e = 50$

$n = 16$

then $m = \cdot 195$ from the table.

$r = e \times m$

$= 50 \times \cdot 195$

$= 9 \cdot 75$

or the slide of the eccentric cutting-frame must be thrown $9\frac{3}{4}$ divisions. (Compare Engleheart, fig. 22.)

RULE 2.—Given radius and number, to find eccentricity.

$$\text{Eccentricity} = \frac{\text{radius}}{\text{modulus}}$$

Example. $r = 2{\cdot}15$

$n = 32$

Then from table $m = {\cdot}097$

$$e = \frac{2{\cdot}15}{.097}$$

$= 22\frac{1}{4}$ about.

Rule 3.—Given radius and eccentricity, to find number.

$$\text{Modulus} = \frac{\text{radius}}{\text{eccentricity}}.$$

When the number can be found from the table, it will generally happen that the modulus, as determined by our rule does not exactly correspond with any value of the modulus given in the table; should this be the case, it indicates that no number of circles that can be obtained by means of the division-plate will exactly form the pattern. Sometimes, when the modulus is small, we can cut the pattern with sufficient accuracy by taking the nearest value of the modulus given in the table.

Example. $r = 12$

$e = 48$

then $m = \frac{12}{48} = {\cdot}25$

This is not a value of modulus given in the table, but we may take ·259 which is taken in the tables, without producing any sensible error, and

$$n = 12.$$

Definition.—When circles are to be arranged in contact with each other and likewise with a given circle whose eccentricity is zero, (or in other words, whose centre is the centre of the work), the latter is called a "guide" circle, which is an "exterior" or "interior" guide circle, according as it lies outside or inside the circles to be arranged.

Definition.—The radius of an exterior guide circle is called the "outer eccentricity," that of an interior guide circle the "inner eccentricity:" these are denoted by e_1 and e_2 respectively. When either the outer or inner eccentricity is given it affords one additional condition for determining the size, number and position of the circles to be cut; and accordingly we can now only take one of the quantities n, e, r, arbitrarily.

RULE 4.—Given outer eccentricity and radius, to determine number and eccentricity.

Eccentricity = outer eccentricity—radius

$$\text{modulus} = \frac{\text{radius}}{\text{eccentricity.}}$$

Example.—Let $e_1 = 60$

$r = 28$

then $e = 32$

$$m = \frac{28}{32} = \cdot 875$$

the nearest tabular value of m is ·866, and as m is large, it will be found that the pattern cannot be cut with the given values of r and e.

RULE 5.—Given outer eccentricity and eccentricity, to determine radius and number.

Radius = outer eccentricity—eccentricity

$$\text{modulus} = \frac{\text{radius}}{\text{eccentricity.}}$$

Example.—Let $e_1 = 100\cdot 5$

$e = 80$

then $r = 20\cdot 5$

$$m = \frac{20\cdot 5}{80} = \cdot 256$$

the nearest tabular value of m is ·259, corresponding to $n = 12$.

Rule 6.—Given outer eccentricity and number, to determine eccentricity and radius.

$$\text{Eccentricity} = \frac{\text{outer eccentricity}}{\text{modulus increased by unity}}$$

$$\text{radius} = \text{outer eccentricity} - \text{eccentricity.}$$

Example.—Let $e_1 = 151$

$n = 12$

then $m = \cdot 259$

$1 + m = 1\cdot 259$

$$e = \frac{151}{1\cdot 259} = 120 \text{ nearly}$$

$\& \; r = 151 - 120 = 31$

Rule 7.—Given inner eccentricity and radius, to determine eccentricity and number.

$$\text{Eccentricity} = \text{inner eccentricity} + \text{radius}$$

$$\text{modulus} = \frac{\text{radius}}{\text{eccentricity.}}$$

Example.—Let $e_2 = 59\frac{1}{2}$

$r = 20\frac{1}{2}$

then $e = 80$

$$m = \frac{20\frac{1}{2}}{80} = \cdot 256$$

$\& \ n = 12$

Rule 8.—Given inner eccentricity and eccentricity, to determine radius and number.

Radius = eccentricity — inner eccentricity

$$\text{modulus} = \frac{\text{radius}}{\text{eccentricity.}}$$

Example.—Let $e_2 = 80$

$e = 60$

then $r = 20$

$$m = \frac{20}{60} = \cdot 333$$

which is most nearly $= \cdot 342$ in the table

and $n = 9$

RULE 9.—Given inner eccentricity and number, to determine eccentricity and radius.

$$\text{Eccentricity} = \frac{\text{inner eccentricity}}{\text{unity diminished by modulus.}}$$

Example.—Let $e_2 = 100$

$n = 20$

then from table $m = \cdot 156$

$$e = \frac{100}{1 - \cdot 156} = \frac{100}{\cdot 844}$$

$$= 118\tfrac{1}{4} \text{ (about)}$$

$$r = 118\tfrac{1}{4} - 100$$

$$= 18\tfrac{1}{4}$$

The rule for placing equal circles in contact with each other, and with an exterior and interior guide circle is

RULE 10.

Radius = half the difference of the outer and inner eccentricities.

Eccentricity = half the sum of the outer and inner eccentricities.

$$\text{Modulus} = \frac{\text{radius}}{\text{eccentricity.}}$$

Example.—Let $e_1 = 100{\cdot}5$

$$e_2 = 59{\cdot}5$$

then
$$r = \frac{100{\cdot}5 - 59{\cdot}5}{2}$$
$$= 20{\cdot}5$$
$$e = \frac{100{\cdot}5 + 59{\cdot}5}{2}$$
$$= 80$$
$$m = \frac{20{\cdot}5}{80}$$
$$= {\cdot}256$$
$$n = 12$$

Rule 11.—The rule for basket-work where equal circles are to be placed in contact with an exterior and interior guide circle, wholly including the latter is

Radius = half the sum of the outer and inner eccentricities.

Eccentricity = half the difference of the outer and inner eccentricities.

Example.—Let $e_2 = 60$

$e_1 = 40$

then $r = \dfrac{60 + 40}{2}$

50

$e = \dfrac{60 - 40}{2}$

$= 10$

3. The mathematician will readily be able to verify the rules above given by observing that the modulus equals the sine of the angle subtended by the radius at the centre of the work; we have

$$m = \sin \frac{\pi}{n}$$

$$r = e\, m$$

$$e = e_1 - r$$

$$e \quad e_2 + r$$

$$e = \frac{e_1 + e_2}{2}$$

$$r = \frac{e_1 - e_2}{2}$$

4. The "Table for Contact of Circles, No. 2"

contains the value of r calculated for a few values of e and n by means of the "TABLE FOR CONTACT OF CIRCLES, No. 1." The first vertical line contains the values of e, the top horizontal line contains the values of n, and the value of r, corresponding to any particular values of e and n, will be found at the intersection of the horizontal line headed by that value of e and of the vertical line headed by that value of n.

Example.—Required the value of r when $e = 60$ and $n = 8$.

We carry the eye along the horizontal line headed 60 and the vertical line headed 8, the number 23 found at their intersection is the value of r required.

It will be observed that all the values of e given in the table have zero in the units' place: so that if the value of e that we employ has a significant digit in the units' place our rule fails because we cannot find e in the first vertical line of the table. When this occurs, calculate the value of r as if the digit in the units' place of e were zero: and add to the result the product of the digit in the units' place of e by the number in the bottom line headed △ of the vertical column headed by the given value of n.

Example.— Required the value of r when $e = 53$ and $n = 6$.

Here the value of r corresponding to $e = 50 = 25$

add 3 × ·5	=	1·5
value of r required	=	26·5

5. The "TABLE FOR CONTACT OF CIRCLES, No. 3," contains the values of e calculated for a few values of r and n by means of the "TABLE FOR CONTACT OF CIRCLES, No. 1." The first vertical line contains the values of r, the top horizontal line contains the values of n, and the value of e corresponding to any particular value of r and n will be found at the intersection of the horizontal line headed by that value of r and of the vertical line headed by that value of n.

Example.—Required the value of e when $r = 32$ and $n = 12$.

Carry the eye along the horizontal line headed 32 till you meet the vertical line headed 12, you find the number 123·52 the value of e required.

6. The tables for the contact of circles can be used not only for the purpose of determining the number and position of circles in contact on the face of the work, but also for determining the number of flutes, beads, or other mouldings that can be placed in contact on the side of the work. Bearing in mind that our calculations have to be verified by trial, we may without much risk of error consider the moulding as made by a semicircular convex or concave tool.

First.—Suppose the tool to be convex, and that it is employed to make flutes.

Suppose that a section of the work be made perpendicular to the lathe axis, the section of each of the flutes will form part of a circle whose radius is half the diameter of the tool, and whose true eccentricity is a very little less than the radius of the work: unless the tool is very large and the diameter of the work very small we may take the eccentricity as being equal to the radius of the work. It must be remembered that in most cases of concave flutes we may use a tool rather too large, and if we do not let it up to cut quite to the depth of its own radius, the two adjacent flutes will leave an edge without diminishing the diameter of the work. Of course if we use too large a tool and let it up to cut to the depth of its own radius we shall slightly diminish the diameter of the work, a result usually of no importance.

As an example. Required to find the breadth of round tool with which to cut a concave moulding on work whose diameter = 180.

Hence $e = \frac{180}{2} = 90$ while we may take any corresponding values of r and n from the tables.

Remember that r = half the diameter of the tool (which is generally marked on it).

Suppose we wish to make 16 flutes.

$$\text{Then } n = 16$$

$$\text{\& from the table } m = \cdot195$$

$$\text{from Rule 1. } r = 90 \times \cdot195$$

$$= 17\cdot55$$

and diameter of the tool required is 35·1

If we have a tool whose diameter is 35, we shall be able to make the cuts without any sensible error, for as our unit is ·01 of an inch the error is only ·001 which may be neglected.

Secondly.—Let the moulding be convex, it will be found that the eccentricity of the semicircular flute is the radius of the work diminished by the radius of the tool. We have, to take our last example—

The radius of the work (R suppose) = 90.

$$e = R - r$$

$$n = 16$$

$$m = \cdot 195$$

$$r = (R - r) \times \cdot 195$$

(From Rule 1.)

$$= R \times \frac{\cdot 195}{1 + \cdot 195}$$

$$= 90 \times \frac{\cdot 195}{1 \cdot 195}$$

$$= 14 \cdot 7 \text{ nearly}$$

and the diameter of the tool should be 29½.

CHAPTER III.

MISCELLANEOUS SIMPLE PATTERNS.

1. If cuts are made with the same radius and eccentricity at all the numbers on the division-plate, or at some of them only taken in regular order, the character of the pattern depends upon whether the radius is greater, equal, or less than the eccentricity.

2. Turks'-heads.—When the radius equals the eccentricity the pattern is called a "Turk's-head;" every circle passes through the centre of the work, and the whole pattern is contained within a circle whose centre coincides with the centre of the work, and whose radius equals twice the eccentricity of each cut.

The varieties of Turks'-heads depend upon the magnitude of r or e, and upon the numbers of the division-plate at which the cuts are made.

The workman may conveniently, in the preparation of the wood, leave a little knob in the centre, as it will be

Plate 1.

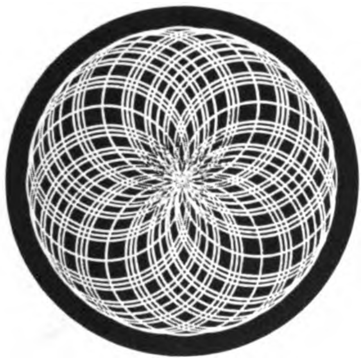
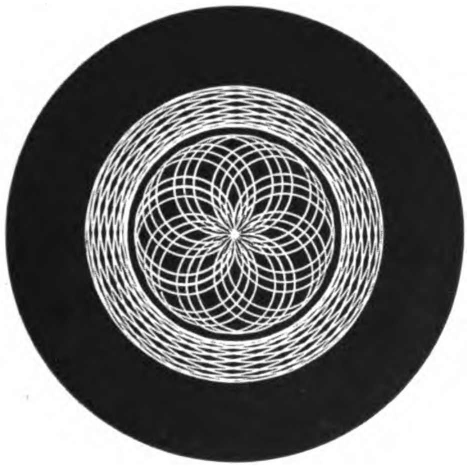


Plate 2.



cut away in the process of cutting the pattern. The most convenient plan to adopt for setting the slide-rest and eccentric cutting-frame is the following :—

Sharpen and polish the tool with which the work is to be cut; put it into the eccentric cutting-frame in the central position, but without taking precautions against loss of time. Let the tool very nearly up to the work by means of the shorter regulating screw of the slide-rest; put on the fluting stops unclamped, let the left-hand stop rest against the slide of the slide-rest, move the slide of the slide-rest forwards till the point of the tool coincides with the centre of the little knob. It is convenient in doing so to keep the slide of the eccentric cutting-frame in a horizontal position. Observe the central reading of the slide-rest, throw it forward to the eccentricity required, clamp the left-hand fluting stop, bring up the right-hand fluting stop and clamp it. Now throw out the slide of the eccentric cutting-frame till upon trial the point of the tool will cut exactly through the centre of the little knob, taking care in making the trials not to hurt the surface of the work. It only remains to cut the pattern at the proper numbers on the division-plate.

A little reflection will show that the reason for making the setting for eccentricity before making that for radius is the following: owing to the operation of the causes above-mentioned (*ante*, p. 16), it is more easy to make the setting for eccentricity with accuracy than it is to make that for radius, and by the artifice of making the radius that with which the tool will cut through the centre of the little knob, we are certain that if the setting for eccentricity is correct, the setting for radius will be correct also.

Although the use of the fluting stops is not absolutely necessary, it is extremely convenient to employ them in cutting any pattern or portion of a pattern where the eccentricity has not to be altered, as the risk of accidentally altering the eccentricity and spoiling the work is thereby obviated.

In the following examples of Turk's-heads I always assume

$$e = 30 = r,$$

though they may be cut with any convenient values of e and r.

PATTERN I.—Cut at every number or at every alternate number on the 96 scale.

Variegated Turks'-heads are made by dividing the total number of possible cuts into groups and omitting similar cuts in each group.

Let it be required, for example, to make a Turk's-head of four similar groups on the scale of 96. Each group consists of 24 cuts, of which some must be omitted.

PATTERN II.—The cuts may be arranged as follows:—

2,	3,	6,	7,	8,	12,	16,	17,	18,	21,	22,	24
26,	27,	30,	31,	32,	36,	40,	41,	42,	45,	46,	48
50,	51,	54,	55,	56,	60,	64,	65,	66,	69,	70,	72
74,	75,	78,	79,	80,	84,	88,	89,	90,	93,	94,	96

PATTERN III.—If it be required to make 6 groups of 16 cuts in each, some of which are to be omitted, we may arrange the cuts as follows :—

1,	4,	5,	9,	10,	11,	16
17,	20,	21,	25,	26,	27,	32
33,	36,	37,	41,	42,	43,	48
49,	52,	53,	57,	58,	59,	64
65,	68,	69,	73,	74,	75,	80
81,	84,	85,	89,	90,	91,	96

The reader can readily make numerous varieties of Turks'-heads for himself.

Perhaps the most convenient method in practice of making a Turk's-head, is to cut all round the division-plate, passing over a certain number of numbers at each cut, and making some of the cuts blind. Thus cuts may be made at every number divisible by 3 on the 96 scale, omitting those divisible by 12. Should the pattern, when complete, appear not to be cut quite finely enough, additional cuts might be made according to some regular rule, thus cuts might be made at the two numbers adjacent to every odd 6, *i.e.* at the numbers 5, 7, 17, 19, &c.

Similarly the cuts might be made at every number divisible by 4, leaving out those divisible by 12 or those divisible by 16.

3. Patterns formed by cutting circles with the same radius bear no distinctive name where the radius is less than the eccentricity, but where the radius is greater than the eccentricity they are called "basket-work."

The beauty of patterns of the first class depends entirely upon some of the circles composing it being placed accurately in contact, and although the final

Plate 3.

adjustment is always effected by trial, the workman will find it convenient to make a preliminary calculation of the values of r, e, n, by the methods given in the preceding Chapter, *ante*, p. 43.

The circles composing basket-work are always intended to be comprised between an outer and inner circle, and the rules for determining their magnitude and position will also be found in the preceding Chapter.

4. SHELLS.—According to the rule generally given for making these patterns the mandrel remains fixed; the radius is diminished by any quantity between any two successive cuts, while the eccentricity is constantly increased or diminished by the same quantity as that by which the radius is diminished.

PATTERN I.—

$$r = 60.$$

Diminish r by 4 and increase e by 4 between each cut, make 14 cuts.

PATTERN II.—

$$r \quad 30.$$

Cut with the following corresponding values of r and e:—

$r=30$	27	25	24	21	19	18	15	13	12	9	7	6	3	1
$e=0$	3	5	6	9	11	12	15	17	18	21	23	24	27	29

Pattern III.—Cut Pattern 1, having adjusted the slide-rest so that it is not quite parallel to the surface of the wood, but a little nearer to it on the left hand.

This may again be varied by making a change of one on the division-plate (scale 144) between each cut after the 2nd ; the slight twist thus produced in the pattern increasing the resemblance to a shell.

5. When a shell is described in the manner above mentioned with the slide-rest in the transverse position, it will point towards or from the centre of the work; if the slide-rest be in the direct position the shell will point in a direction parallel to the axis of the work; and if the slide-rest be in an oblique position the shell will point along a horizontal line parallel to the slide-rest. There is, however, in cases where the outer circle of the shell is eccentrical, another way in which it may be described, viz., by diminishing the radius by a certain quantity, and by turning the division-plate through a certain number

PLATE 4.

of divisions between each cut. Although the number of divisions through which the division-plate has to be turned can be ascertained by trial, it appears convenient to calculate the number. This may be done in the manner following, in those cases at least in which the outer circles of the shells are in contact.

Let us suppose that there are six shells whose outer circles are in contact, these circles can be made by cutting with radius 30 and eccentricity 60 at the numbers

96, 16, 32, 48, 64, 80.

It follows that a movement of the division-plate through sixteen divisions will move the tool from the position in which it can cut the outer circle of one shell to that in which it can cut the outer circle of the adjoining shell; a movement of half that number or eight divisions would put it into the position for cutting a circle through the centres of these two circles, or if the radius became zero would enable it to cut a dot on the circumference of one of the circles, a cut which, though it would be omitted in practice, is in theory the last cut of the shell. We find, therefore, that we must diminish the radius by $\frac{30}{8}$ $= 3\frac{3}{4}$, corresponding to a change either backwards or forwards of one division on the division-plate. Hence if we wish to make all the shells point in the same direction, considered with respect to their positions in the circle, we cut the second circles of the shells at

1, 17, 33, 49, 65, 81,

with radius $30 - 3\frac{3}{4} = 26\frac{1}{4}$; we must proceed in the same manner for the rest of the circles, increasing the numbers on the division-plate by unity and diminishing the radius by $3\frac{3}{4}$ between each cut. If we wish to make the alternate shells point in opposite directions we must increase the numbers

96, 32, 64,

and diminish the numbers

16, 48, 80

by unity for the second cut, and proceed in a similar manner till the shells are complete.

If we think that the change of radius between each cut is too great, we must cut the shells on some other scale of the division-plate, for example we might cut at

120, 20, 40, 60, 80, 100

on the scale of 120, the change of radius between each cut would be $\frac{30}{10}$ or 3. We might even use the scale of 360, when we should cut at the numbers

360, 60, 120, 180, 240, 300,

and the change of radius between two consecutive cuts would be $\frac{30}{30}$ or 1. The rule being that the diminution in the value of r corresponding to a change of one division on the division-plate equals $\frac{2\,r}{n}$ or twice the quotient of the radius divided by the number of divisions passed over in passing from the outer circle of one shell to the outer circle of the next shell. The reader will be easily able to devise for himself patterns consisting of variegated shells arranged in this manner.

Pattern IV.—With $r = 30$, $e = 60$, cut on the scale of 360 at

360, 60, 120, 180, 240, 300,

with $r = 29$ cut at

359, 61, 119, 181, 239, 301,

with $r = 27$ cut at

357, 63, 117, 183, 237, 303,

with $r = 24$ cut at

354, 66, 114, 186, 234, 306,

with $r = 20$ cut at

350, 70, 110, 190, 230, 310,

with $r = 19$ cut at

349, 71, 109, 191, 229, 311,

with $r = 17$ cut at

347, 73, 107, 193, 227, 313,

with $r = 14$ cut at

344, 76, 104, 196, 224, 316,

with $r = 10$ cut at

340, 80, 100, 200, 220, 320,

with $r = 9$ cut at

339, 81, 99, 201, 219, 321,

with $r = 7$ cut at

337, 83, 97, 203, 217, 323.

with $r = 4$ cut at

334, 86, 94, 206, 214, 326.

The method here pointed out can be employed with the slide-rest in any position, so long as it is parallel to the face of the work operated on. It is, perhaps, necessary to add that if it be not in the transverse position we must, in making the calculations for placing the outer circles of the shells in contact, take for "the eccentricity" the true eccentricity, *i.e.*, true distance of the centre of the outer circle of the shell from the axis of the work.

6. I have considered it unnecessary to give the settings of any patterns consisting of basket-work, or of circles in contact, as many beautiful examples will be found in Engleheart's "Eccentric Turning." It may perhaps be convenient to give one or more skeleton outlines of patterns, the details of which can be filled up according to the taste of the operator.

We generally wish to make the pattern as large as we can, having regard to the general appearance of the work; if the work be of moderate size, say having a diameter of about 150, we should leave a margin of at least 10; a larger margin should be left if the work be larger.

We generally take a certain number of circles in contact as the basis of the pattern; and it will be observed that if we wish to make the pattern as large as possible, we have first to determine their eccentricity and radius as determined by their touching an exterior guide circle, viz., the inside of the margin that is to be left.

Let the radius of the work $= R$
width of margin $= M$
then "the outer eccentricity," $e_1 = R - M$

and from Rule 6 we have—

$$e = \frac{e_1}{1 + m}$$

$$r = e_1 - e$$

where m is the modulus corresponding to the given number of circles.

Example.—Let it be required to describe the largest four circles that can be placed in contact on a draughtsman whose radius = 70.

Here $R = 70$
Let $M = 10$
Then $e_1 = 70 - 10 = 60$.
From Table (1) $m = \cdot 707$

$$e = \frac{60}{1 \cdot 707}$$

$$= 35 \cdot 1$$

$$r = 60 - 35 \cdot 1$$

$$= 24 \cdot 9$$

We may take $e = 35$
$r = 25$

without any sensible risk of error.

We can now cut a great variety of patterns. We may fill in the four circles with shells, either by increasing or decreasing the eccentricity between each cut, or we may superimpose another series of four circles equidistant between the original four, but cutting at the numbers

96, 12, 24, 36, 48, 60, 72, 84,

and these circles may be filled up with shells in a similar manner. Compare Engleheart, figures 6 & 7.

A good pattern may be made by filling up the circles with concentric circles, using a flat-ended tool, diminishing the radius between each cut by a quantity equal to the width of the tool, cutting the first circles very deep and diminishing the depth of cut by an equal quantity between each cut.

Example 2.—Let it be required to cut a border on work whose diameter is 200, round an interior pattern which extends to a distance 60 from the centre of the work.

Here $R = 100$

$M = 15$ say,

and we have to cut our pattern between two guide circles,

$$e_1 = 100 - 15$$
$$= 85$$
$$e_2 = 60$$

and from Rule 10 we have—

$$r = \frac{85 - 60}{2} = \frac{25}{2}$$
$$= 12{\cdot}5$$
$$e = \frac{85 + 60}{2} = \frac{145}{2}$$
$$= 72{\cdot}5$$
$$m = \frac{r}{e} = \frac{12{\cdot}5}{72{\cdot}5}$$
$$= {\cdot}172$$

Whence $n = 18$

We can, as in the former example, fill up the circles with shells: but in this case the better way will be to repeat the circles so as to form a border by cutting at every even number on the scale of 144.

Example 3.—The border in the last example may be filled in with basket-work instead of with circles in contact.

As before

$$e_1 = 85$$
$$e_2 = 60$$

and from Rule 11

$$r = \frac{85 + 60}{2}$$
$$= 72 \cdot 5$$
$$e = \frac{85 - 60}{2}$$
$$= 12 \cdot 5$$

7. No definite rule can be given as to the number of circles that should be cut in a basket-work pattern. The author generally commences by making cuts at the numbers that correspond to the foundation cut of the pattern that has been described inside the basket. If, for example, the foundation of the interior pattern was 8 circles cut at

96, 12, 24, 36, 48, 60, 72, 84,

the workman should make the first cuts of his basket at those numbers, and then make intermediate cuts at the numbers 6, 18, &c.; an inspection of the pattern would then show if it was desirable to make more intermediate cuts, and if so whether it was desirable to cut at 6, 9, &c., (after which no further cuts could be made equidistant from each other, unless a cut were made at each number,) or at 2, 4, 8, 10, &c.

The author has made no experiments in variegated basket-work ; in cutting it the workman might adopt the same method as he would in cutting a variegated Turk's-head, remembering, however, that fewer cuts would be necessary.

When basket-work is cut with a "left-side" tool, *i.e.*, one which has a cutting edge towards the left, a very beautiful effect is produced.

CHAPTER IV.

PART I.

THE THEORY OF DUAL COUNTING.

1. Chuck a piece of boxwood; place the slide-rest in the transverse position, surface the wood; draw with a pencil any curve, straight line, or combination of curves and straight lines, on the surface of the wood; place the eccentric cutting-frame containing a tool in the central position in the tool-holder of the slide-rest. It will be found that by fixing the index in the proper hole on the division-plate, and by moving the slide of the slide-rest into the proper position, the eccentric cutting-frame can be brought into such a position as to enable a dot to be cut on the pencil line. Now placing the index in the next hole on the division-plate, it will be found that by making an appropriate movement of the slide-rest, the tool can be made to cut another dot on the pencil line; and proceeding in this manner it will be found that the

whole of the pencil line can be covered with dots. There are two exceptional cases: *first*, where the line drawn by the pencil is a straight line passing though the centre of the work; in this case the index must be kept in the same hole on the division-plate while the slide-rest is moved: *secondly*, where the line drawn by the pencil is a circle of no eccentricity; here the index must be moved, and the slide-rest retained in the same position. Subject to these exceptions, we shall be correct in saying that by moving the division-plate and slide-rest in the appropriate manner, we can cut dots on any curve, straight line, or combination of curves and straight lines; and if, instead of cutting dots, we throw out the slide of the eccentric cutting-frame, we shall produce a pattern cut in circles.

Although this method of producing a pattern must have suggested itself to every turner, the author has not seen many patterns produced by it; one or two will be found in Englcheart's "Eccentric Turning," made on this principle, though, in the directions there given for cutting them, the eccentric chuck is employed. It appears convenient to have a name for the method: the author proposes to call it "dual counting."

2. The mathematician will observe that the process of dual counting consists in cutting a curve by means of its polar coordinates, the eccentricity corresponding to the radius vector, and the angle through which the division plate has been moved to the anomaly of any point of the curve. It follows that the cuts can be arranged on any curve of which the equation is known, the only difficulty in doing so being the labour of calculating the eccentricities corresponding to each number on the division-plate.

The author has calculated and tabulated the eccentricities for each cut on the curve whose equation is

$$r = a + b \sin m\,\theta$$

for a few values of b and m. The method of using the tables containing the results of these calculations will be found in Parts II. and III. of this present chapter; the corresponding patterns consist of looped and waved figures, eccentrical circles passing through the centre of the work, and the cardioid. He has also calculated and tabulated the eccentricities for each cut on the straight line

$$r = a + b \sec \theta.$$

The method of using the tables containing the results of these calculations will be found in Part IV. of the present chapter. He has also calculated the eccentricities for each cut on the ellipse

$$r = \frac{a\sqrt{1 - l^2}}{\sqrt{1 - l^2 \cos^2 \theta}}$$

where l = the "eccentricity" of the ellipse.

See Part V. of the present chapter.

There is no difficulty in calculating the eccentricities for either of the first mentioned curves, as the values of sin. $m\,\theta$ and sec. θ can be taken without difficulty from the trigonometrical tables.

The calculation of the eccentricities of each cut of the ellipse can be put under the form following :—

$$\text{Log (eccentricity)} = \log a + \log \cos \zeta - \log \cos \phi$$

[where $l = \sin \zeta$ and $l \cos \theta = \sin \phi$,

where $\log a + \log \cos \zeta$ is constant for any particular ellipse. Should the semiaxis minor be given instead of l, we determine l by the formula

$$l = \sin \psi$$

$$\text{where } \log \cos \psi = \log a - \log b.$$

3. When the reader has read this chapter and has cut some of the patterns according to the methods here given, he will be able to draw a comparison between dual counting and double counting.

First, let us consider the number of adjustments required to cut a pattern by either method.

Whichever method we employ, we have, if we use the scale of 96, to make 96 adjustments of the division-plate. As the number of divisions passed over between each cut in double counting is the same, while this is not the case in dual counting, there appears to be a slight advantage so far as the motion of the division-plate is concerned in favour of double counting, but we may fairly set against this the advantage possessed by dual counting of having the numbers employed printed in tables which can be laid on the lathe under the eye of the workman.

In double counting we must make one adjustment of the eccentric chuck corresponding to each cut, *i.e.*, 96 or 120 adjustments, according to the scale of the division-plate that we employ. In dual counting we have to make one adjustment of the slide-rest for several cuts on the division-plate ; in an ellipse, for instance, we have to make 24 adjustments only of the slide-rest for the 96 cuts which complete the pattern, while in double counting we should have had to have made 96 adjustments of the eccentric chuck. It will be observed that, in the vast majority of the patterns, only a few adjustments have to be made; and on the whole it appears that the advantage in point of trouble, and therefore in accuracy, will be found in favour of dual counting.

Lastly, by dual counting we are able, if we like to make the calculations, to arrange the cuts on any curve whatever, while in double counting we are restricted to those curves which are produced by two circular motions.

PART II.

LOOPED FIGURES AND THEIR DERIVATIVES.

4. The reader will probably have at first some little difficulty in understanding the directions here given for cutting looped figures. He is therefore advised, after studying them, to turn to the "Settings for 4-looped Figures," patterns 1 and 2, and to the "Settings for 5-looped Figures," patterns 7 and 4, at pages 141 and 134, and to write down exactly what cuts he considers that he ought to make according to the directions there given, and with what eccentricities he ought to make them. He will find the calculations for the eccentricities of the cuts in these patterns worked out at length at page 78, *et seq.*

5. Two "Tables of Settings" (*post*, p. 188 to 199) are given for each variety of looped figures here treated of: No. I., headed "DIVISION-PLATE SCALE 120" (or whatever the scale may be), contains all the numbers in the particular scale mentioned, arranged in vertical columns

denoted by the numbers 1, 2, 3, &c., and in horizontal lines denoted by the letters A, B, C, &c.; No. II., headed "DIFFERENCES OF ECCENTRICITY" contains the necessary changes of eccentricity arranged in vertical columns, denoted by the numbers 100, &c., and in horizontal lines denoted by the letters *A*, *B*, &c.

6. In the directions for cutting a pattern, any one of the letters A, B, &c., standing alone, or separated from the following letters by a comma (,), means that the workman is to "make cuts with the same radius and eccentricity at all the numbers on the division-plate that occur in the table, in the same horizontal line as such letter." For example, in the directions for cutting 5-looped figures, the letter A means "Make cuts with the same radius and eccentricity at the numbers

6, 30, 54, 78, 102,

on the scale of 120."

Any two or more of the letters AB, &c. occurring together, not separated from each other by a comma or full stop, mean "Make cuts with the same radius and eccentricity at all the numbers that occur in the table in the same horizontal lines as such letters." For example, in the directions for cutting five looped figures, A G N means "make cuts with the same radius and eccentricity at the numbers

6 30 54 78 102
120 12 24 36 48 60 72 84 96 108
18 42 66 90 114

on the scale of 120."

7. A comma denotes a change of eccentricity. Unless it is otherwise stated, this change is a diminution of eccentricity, that is, the screw of the slide-rest is to be moved backwards so as to carry the tool-holder towards the right of the work.

The successive changes of eccentricity in the pattern will be found in the Table No. II., headed "DIFFERENCES OF ECCENTRICITY." The successive changes are always taken from the same vertical column, beginning at a number opposite a named letter *A*, *B*, &c., and reading down the column; the particular column to be employed is headed by the number given in the direction "$b = 60$," or whatever the number may be.

For example, let the direction in the settings for 5-looped figures be

$$b = 30 \text{ from } G.$$

We turn to the column headed 30 in the Table of DIFFERENCES OF ECCENTRICITY for 5-looped figures, and reading down the column beginning with the number opposite *G* we have for the successive changes of eccentricity

$$7\tfrac{3}{4},\ 7\tfrac{1}{4},\ 6\tfrac{1}{4},\ 4\tfrac{3}{4},\ 3,\ 1.$$

8. The direction

"$e = 100$"

means that the eccentricity for the first cut of the pattern, or part of the pattern, is 100, or ten whole turns forward of the slide-rest (*ante*, p. 19). The direction

"$e = 100$ to 30," or "$e = 100$ or 30"

means that any value of *e*, from 100 to 30 inclusive, may be taken for the first cut.

9. A full stop occurring after a letter means that the pattern, or part of the pattern, is finished. In the latter case, the eccentricity for the following cut (if any) will be obtained either from the preceding or from the following rule.

10. A letter, or letters, appearing within braces, thus,

[A] or [D K]

means that the eccentricity with which it, or they, is or are to be cut is the eccentricity for the next cut of the pattern. In this case, the same letter or letters will be repeated at the commencement of the next part of the pattern, where it is to be cut *blind*, and only serves as the starting-point whence the changes of eccentricity are to be calculated for that part of the pattern.

11. The direction (*ante*, p. 19)

$$r = 10$$

means that the radius is ·1 of an inch, or that the tool-holder of the eccentric cutting-frame is to be thrown out one complete turn of the screw-head from the central position.

12. It will sometimes happen that the successive changes of eccentricity are taken together greater than the eccentricity of the first cut, or, in other words, that the pattern crosses the centre of the work. This will be indicated by our having to diminish the eccentricity of some particular cut by a number greater than itself. In

order to obviate the inconvenience which thus arises, it is convenient, in making the calculations, to add 100 to the apparent eccentricity of the cut where this occurs without moving the slide-rest screw forward.

EXAMPLES.

Here follow the calculations for producing some of the patterns worked out at length. The reader is recommended to endeavour to make the calculations for himself, from the settings and the Tables, and only to look at the calculations here worked out for the purpose of verifying his results.

Example 1.—The first pattern in the "Settings for 4-looped Figures," is

$r = 10 \qquad e = 100 \qquad b = 30$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

According to our rules, this becomes, when translated into ordinary language—

"Throw out the slide of the eccentric cutting-frame one whole turn of the screw from the central position of the tool, throw out the slide of the slide-rest ten whole turns of the screw from the position all at centre; or rather, having regard to the danger of loss of time, throw it out 10½ turns and then turn it back half a turn. Cut at the following numbers on the division-plate (scale 96)—

6 30 54 78;

diminish eccentricity by 1, cut at

5 7 29 31 53 55 77 79;

diminish eccentricity by 3, cut at

4 8 28 32 52 56 76 80;

diminish eccentricity by 4¾, cut at

3 9 27 33 51 57 75 81;

diminish eccentricity by 6¼, cut at

2 10 26 34 50 58 74 82;

diminish eccentricity by 7¼, cut at

1 11 25 35 49 59 73 83;

diminish eccentricity by 7¾, cut at

96 12 24 36 48 60 72 84;

diminish eccentricity by 7¾, cut at

95 13 23 37 47 61 71 85;

diminish eccentricity by 7¼, cut at

94 14 22 38 46 62 70 86;

diminish eccentricity by $6\frac{1}{4}$, cut at

93 15 21 39 45 63 69 87;

diminish eccentricity by $4\frac{3}{4}$, cut at

92 16 20 40 44 64 68 88;

diminish eccentricity by 3, cut at

91 17 19 41 43 65 67 89;

diminish eccentricity by 1, cut at

18 42 66 90."

Although it is possible to cut the patterns by mere inspection of the Table, changing the eccentricity by the tabular number after cutting each row of figures, there is a considerable risk of error in doing so. The reader is therefore strongly advised to calculate the apparent eccentricity for each cut before he begins to work. The calculations may be conveniently arranged as follows, where I assume the central reading of the slide-rest to be $4\frac{1}{4}$, and therefore the apparent eccentricity of the first cut to be $104\frac{1}{4}$:—

A	$104\frac{1}{4}$
	1
B	103
	3
C	$100\frac{1}{4}$

D	$\frac{4\frac{3}{4}}{95\frac{1}{2}}$
E	$\frac{6\frac{1}{4}}{89\frac{1}{4}}$
F	$\frac{7\frac{1}{4}}{82}$
G	$\frac{7\frac{3}{4}}{74\frac{1}{4}}$
H	$\frac{7\frac{3}{4}}{66\frac{1}{2}}$
J	$\frac{7\frac{1}{4}}{59\frac{1}{4}}$
K	$\frac{6\frac{1}{4}}{53}$
L	$\frac{4\frac{3}{4}}{48\frac{1}{4}}$
M	$\frac{3}{45\frac{1}{4}}$
N	$\frac{1}{44\frac{1}{4}}$

The numbers opposite to each of the numbers A, B, &c., are the apparent eccentricities with which cuts are to be made at the corresponding numbers. The number opposite A is obtained by adding the central reading $4\frac{1}{4}$ to 100, the eccentricity given for the first cut: the succeeding apparent eccentricities are obtained by subtracting the successive numbers in the column headed 30 in the Table of "DIFFERENCES OF ECCENTRICITY," from each preceding apparent eccentricity.

Example 2.—The directions given for cutting Pattern 7 (in the Patterns for 5-looped figures) are—

$r = 10 \qquad e = 100 \qquad b =$ 50 from A.

A, B, C, [DK], E, F,

$b =$ 50 from A.

DK, JL, HM, N.

The above directions, in ordinary language, become: "With radius 10, and eccentricity 100, make cuts at the following numbers on the division-plate (scale 120)—

6 30 54 78 102;

diminish eccentricity by $1\frac{1}{2}$, and cut at the numbers

5 7 29 31 53 55 77 79 101 103;

diminish eccentricity by $5\frac{1}{4}$, and cut at the numbers

4 8 28 32 52 56 76 80 100 104;

diminish eccentricity by 8, and cut at the numbers

3 9 27 33 51 57 75 81 99 105

117 15 21 39 45 63 69 87 93 111;

diminish eccentricity by $10\frac{1}{4}$, and cut at the numbers

2 10 26 34 50 58 74 82 98 106;

diminish eccentricity by 12, and cut at the numbers

1 11 25 35 49 59 73 83 97 107;

Increase the eccentricity, until the tool will fit exactly into the cuts at the numbers 3 9 27 33 51, &c.

diminish eccentricity by $1\frac{1}{2}$, and cut at the numbers

118 15 22 38 46 62 70 86 94 110

116 16 20 40 44 64 68 88 92 112;

diminish eccentricity by $5\frac{1}{4}$, and cut at the numbers

119 13 23 37 47 61 71 85 95 109

115 17 19 41 43 65 67 89 91 113;

diminish eccentricity by 8, and cut at the numbers

18 42 66 90 114."

The calculations may be conveniently arranged in the following manner (where I assume the "central reading" of the slide-rest to be $6\frac{1}{2}$, and therefore the apparent eccentricity of the first cut to be $106\frac{1}{2}$)—

A	$106\frac{1}{2}$		
	$1\frac{1}{2}$		
B . . .	105		
	$5\frac{1}{4}$		
C	$99\frac{3}{4}$		
	8		
DK	$91\frac{3}{4}$	DK	$91\frac{3}{4}$
	$10\frac{1}{4}$		$1\frac{1}{2}$
E	$81\frac{1}{2}$	JL	$90\frac{1}{4}$
	12		$5\frac{1}{4}$
F	$69\frac{1}{2}$	HM	85
			8
		N	77

It will be observed that the eccentricities for JL and HM, are greater than that of E: it is therefore advisable for the sake of avoiding loss of time to cut them before cutting E, for a similar reason N should be cut before F.

Example 3.—Let the directions for cutting a pattern in 4-looped figures be—

$r = 10 \qquad e = 100 \qquad b = 60$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

These become, in ordinary language: "With radius 10, and eccentricity 100, cut at the following numbers on the division-plate (scale 96)—

6 30 54 78;

diminish eccentricity by 2, cut at

5 7 29 31 53 55 77 79;

diminish eccentricity by 6, cut at

4 8 28 32 52 56 76 80;

diminish eccentricity by 9½, cut at

3 9 27 33 51 57 75 81;

diminish eccentricity by 12½, cut at

2 10 26 34 50 58 74 82;

diminish eccentricity by 14½, cut at

1 11 25 35 49 59 73 83;

diminish eccentricity by 15½, cut at

96 12 24 36 48 60 72 84;

diminish eccentricity by 15½, cut at

95 13 23 37 47 61 71 85;

diminish eccentricity by 14½, cut at

94 14 22 38 46 62 70 86;

diminish eccentricity by 12½, cut at

93 15 21 39 45 63 69 87;

diminish eccentricity by $9\frac{1}{2}$, cut at
92 16 20 40 44 64 68 88;
diminish eccentricity by 6, cut at
91 17 19 41 43 65 67 89;
diminish eccentricity by 2, cut at
18 42 66 90."

The calculations may be arranged as follows: assuming the central reading to be $\frac{1}{2}$ we have—

A	$100\frac{1}{2}$
	2
B	$98\frac{1}{2}$
	6
C	$92\frac{1}{2}$
	$9\frac{1}{2}$
D	83
	$12\frac{1}{2}$
E	$70\frac{1}{2}$
	$14\frac{1}{2}$
F	56
	$15\frac{1}{2}$
G	$40\frac{1}{2}$
	$15\frac{1}{2}$
H	25

	14½	
J	10½	110½
	12½	12½
K		98
		9½
L		88½
		6
M		82½
		2
N		80½

Where we write 110½ instead of 10½ for J (owing to rule given in Art. 12, ante, p. 77) making no change in the position of the slide-rest.

Example 4.—The directions for cutting Pattern 11 (in the 5-looped figures) are—

$r = 10 \qquad e = 100 \qquad b = 50$ (or 30) from A.

DK, CEJL, BFHM, AGN.

This is, in ordinary language: "With radius 10, and eccentricity 100, cut at the following numbers of the division-plate (scale 120)—

3	9	27	33	51	57	75	81	99	105
117	15	21	39	45	63	69	87	93	111 ;

diminish eccentricity by $1\frac{1}{2}$, taking $b = 50$, cut at

4	8	28	32	52	56	76	80	100	104
2	10	26	34	50	58	74	82	98	106
118	14	22	38	46	62	70	86	94	110
116	16	20	40	44	64	68	88	92	112 ;

diminish eccentricity by $5\frac{1}{4}$, cut at

5	7	29	31	53	55	77	79	101	103
1	11	25	35	49	59	73	83	97	107
119	13	23	37	47	61	71	85	95	109
115	17	19	41	43	65	67	89	91	113 ;

diminish eccentricity by 8, cut at

6		30		54		78		102	
120	12	24	36	48	60	72	84	96	108
18		42		66		90		114."	

For the centre, repeat making the first cut with eccentricity 40.

The calculations may be arranged as follows. A little consideration of the nature of the pattern, will show that a small variation of the eccentricity of the first cut will not signify much, and accordingly we may neglect the central reading, and may work with tabular instead of apparent eccentricities.

For Border Pattern.		For Centre.	
DK	100	DK	40
	$1\frac{1}{2}$		
CEJL	$98\frac{1}{2}$	CEJL	$38\frac{1}{2}$
	$5\frac{1}{4}$		
BFHM	$93\frac{1}{4}$	BFHM	$33\frac{1}{4}$
	8		
AGN	$85\frac{1}{4}$	AGN	$25\frac{1}{4}$

This pattern is easy to cut, is very beautiful, and is of very great utility; the border looks extremely well cut on work not quite perpendicular to the axis of the lathe; in other words, when cut with the slide-rest in an oblique position.

Example 5.—The directions for cutting Pattern 14 (in the 5-looped figures) are—

$$r = 10 \qquad e = 100 \qquad b = 50 \text{ from } A.$$

K, JL, HM, [GN],

cutting at the numbers in the even columns only (except GN).

b = 5O from G.

GN, F, E, D, C, B, A.

b = 5O from K.

GN, HM, JL, K,

cutting at the numbers in the odd columns only.

This becomes, in ordinary language: "With radius and eccentricity 100, cut at

15 39 63 87 111;

diminish eccentricity by $1\frac{1}{2}$, cut at

14 38 62 86 110

16 40 64 88 112;

diminish eccentricity by $5\frac{1}{4}$, cut at

13 37 61 85 109

17 41 65 89 113;

diminish eccentricity by 8, cut at

120 12 24 36 48 60 72 84 96 108

18 42 66 90 114;

diminish eccentricity by 13, cut at

1 11 25 35 49 59 73 83 97 107;

diminish eccentricity by 12, cut at

2 10 26 34 50 58 74 82 98 106;

diminish eccentricity by $10\frac{1}{4}$, cut at

3 9 27 33 51 57 75 81 99 105;

diminish eccentricity by 8, cut at

4 8 28 32 52 56 76 80 100 104;

diminish eccentricity by $5\frac{1}{4}$, cut at

5 7 29 31 53 55 77 79 101 103;

diminish eccentricity by $1\frac{1}{2}$, cut at

6 30 54 78 102.

Replace the cutting-frame to the position in which it was when you cut 120, &c.—

diminish eccentricity by 8, cut at

119 23 47 71 95

115 19 43 67 91;

diminish eccentricity by $5\frac{1}{4}$, cut at

118 22 46 70 94

116 20 44 68 92;

diminish eccentricity by $1\frac{1}{2}$, cut at

117 21 45 69 93."

The calculations may be arranged as follows: let the central reading = $5\frac{1}{2}$, we have

K	$105\frac{1}{2}$
	$1\frac{1}{2}$
JL	104

	5¼		
HM	98¾		
	8		
GN	90¾	GN	90¾
	13		8
F	77¾	HM	82¾
	12		5¼
E	65¾	JL	77½
	10¼		1½
D	55½	K	76
	8		
C	47½		
	5¼		
B	42¼		
	1½		
A	40¾		

It will be observed that, in order to avoid loss of time, all the cuts in the odd columns of HM, JL, K, the eccentricities of which are given on the right-hand side above, and lie between those of GN and F, should be made after GN and before F.

13. In cutting a pattern, the author adopts the following practice: he makes the calculations in the manner already shewn, on a piece of paper, which he

places on the lathe; he has the Table headed "DIVISION-PLATE, SCALE —" (copies of which, for use on the lathe, will be found in the pocket of the cover to this book), which is required for the pattern open before him; when he has set the eccentric cutting-frame and the slide-rest for the first cut, he lays a ruler on the Table immediately below the line of figures, A for instance, which he is going to cut, so as to guide his eye; when he has cut it, he draws a pencil through A on his calculations, and having moved the slide-rest screw, if there is a change of eccentricity, he places the ruler immediately below the next line of figures to be cut.

14. An examination of the first three patterns given in the settings for five- and four-looped figures shews that when all the cuts given in the Table are cut in succession with $b =$ (whatever the number may be) from A, the curve on which lie the centres of the circles forming the pattern, has the following properties:

If $2b$ be less than the eccentricity of the first cut, the curve never reaches the centre of the work, and is composed of parts alternately concave and convex: the convex and concave parts can be separated by a circle described with $e = 0$, $r = b$.

It may perhaps be remarked, that if e be taken very large indeed with respect to b, the curve becomes, in theory, entirely convex, with convexity alternately increasing and diminishing: on the scale on which we work, on the lathe, this effect will never become apparent.

If $2b =$ the eccentricity for the first cut, the curve exactly reaches the centre, giving rise to loops in number 5, 4, &c.

If $2b$ be greater than the eccentricity for the first cut, the curve passes beyond the centre, giving rise to double the number of loops. But if the original number of loops was odd, the loops produced by the branches of the curve on opposite sides of the centre lie on each other, and if b equals the eccentricity for the first cut, they are coincident.

15. The numbers given in the Tables in any column $b = 100, 80$, &c., are the same read upwards or downwards; their sum equals $2b$.

The patterns are produced by making cuts on the curve, or on parts of the curve mentioned in the last article; and by repeating such cuts on the same curve, or part of a curve turned round through an angle.

16. The mathematician will readily deduce all the properties of the curve on which the cuts lie, from the following considerations—

Let e and θ be its polar co-ordinates,
then $e = a + b \sin. m\,\theta$

where a and b are arbitrary constants, m is constant for the table of m-looped figures.

Giving to sin. $m\,\theta$ its maxima and minima values, we have

$$e_1 = a + b$$
$$e_2 = a - b$$
$$\text{whence } e_2 = e_1 - 2\,b$$
$$\text{or } e_2 > = \text{ or } < 0 \text{ as}$$
$$e_1 > = \text{ or } < 2\,b$$

The numbers given in each of the columns $b = 100$, 80, &c., in the Tables of DIFFERENCES OF ECCENTRICITY are the differences of sin. $m\,\theta$, where θ is the angle corresponding to two consecutive numbers on the division-plate, multiplied by the number at the head of the column.

PART III.

CIRCULAR FIGURES AND THEIR DERIVATIVES.

17. THE figures produced by the method given in this part of the chapter, are perhaps the most remarkable, though not the most beautiful, of those described in this book. They are intended to be cut with the slide-rest in the transverse position, and they consist, with but one exception, entirely of figures in which the cuts are arranged on circles which pass through the centre of the work; although most of these figures have been cut before, they have not, to the author's knowledge, been cut without the aid of an eccentric chuck. The author considers it unnecessary to give more than one example worked out at length, as the attentive reader will have but little difficulty in understanding the notation.

18. The Tables for Circular Figures (*post,* p. 200, *et seq.*) consist of four Tables. Each of the first three Tables, Nos. I., II., and III., headed DIVISION-PLATE, SCALE 96, is divided into pairs of vertical columns, denoted by

the numbers I., II., &c.; the separate columns in each pair being denoted by the figures 1, 2, and into horizontal lines denoted by the letters A, B, &c. If cuts are made at all the numbers in any one pair of the vertical columns, with the proper eccentricities, they will produce a circle passing through the centre of the work; the cut at A being made with slide-rest in its central position, and the cut at Z lying at the opposite extremity of that diameter of the circle which passes through the centre of the work, or as I propose to call it, the *central* diameter.

The central diameters of the several circles produced by cutting at the pairs of columns III., V., VII. in TABLE I., form angles of 90°, 180°, and 270° respectively, with the central diameter of the circle produced by cutting at the pair of columns I. in the same Table. The circles produced by cutting at the vertical columns in TABLE II. are similarly situated with respect to each other, but occupy intermediate positions with respect to the circles produced by TABLE I.; so that, if all the circles are cut from TABLE I. and TABLE II., we obtain eight circles so arranged that the central diameters of any two adjacent circles contain an angle of 45°.

The central diameters of the circles produced by cutting at any two adjacent columns of TABLE III., contain an angle of 60°.

The remaining TABLE FOR CIRCULAR FIGURES is headed ECCENTRICITIES, and contains the eccentricities with which the cuts are to be made. The Table is arranged in vertical columns, denoted by the numbers 100, 80, &c., which denote the diameter of the circle that will be formed when the eccentricities in that column are employed, and in horizontal lines denoted by the letters *A*, *B*, &c.

19. In the directions for cutting a pattern, the particular column in the TABLE OF ECCENTRICITIES that is to be employed is denoted by the direction "$d = 100$," or whatever the number may be. Thus the direction

$$d = 30$$

means that the eccentricities for the successive cuts are to be taken from the column headed 30: it also indicates that the cuts will lie on a circle whose diameter equals 30

20. In the directions for cutting a pattern, any one of the letters A, B, C, &c. standing alone, or separated

from the following letters by a comma, means that the workman is to "Make cuts with the same radius at all the numbers that occur in the Table in the same horizontal line as such letter, with the eccentricity denoted by the corresponding italic letter *A*, *B*, or *C*, as the case may be, in the TABLE OF ECCENTRICITIES."

For example, if we are using TABLE I., and the direction $d = 30$ be given, the letter D means, "Make cuts with the same radius at the numbers

3, 45, 27, 69, 51, 93, 75, 21,

with the eccentricity $9\frac{3}{4}$."

Any two or more of the letters A, B, C, &c. occurring together, not separated by a comma or full stop, mean, "Make cuts with the same radius at all the numbers that occur in the same horizontal lines as such letters, with the eccentricity corresponding to the first of them."

For example, if we are using TABLE II., and the direction $d = 30$ be given, the letters E G, mean, "Make cuts with the same radius at the numbers

16, 56, 40, 80, 64, 8, 88, 32,
18, 54, 42, 78, 66, 6, 90, 30,

with the eccentricity $5\frac{3}{4}$, which corresponds to *E*;" while if the letters had been G E, we should have cut at the same numbers with the eccentricity $11\frac{1}{2}$, which corresponds to *G*.

For brevity I sometimes use the notation "A to Z," meaning make cuts at

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z;

or "D to K," meaning make cuts at

D, E, F, G, H, J, K.

21. A comma denotes a change of eccentricity. As we work from a Table of the eccentricities themselves, we shall sometimes find it convenient to make the first cut at the outside of the pattern; *i. e.* as far as possible from the centre of the work; and to make the successive changes of eccentricity by turning the screw of the slide-rest backwards, and at other times we shall find it convenient to make the first cut at the inside of the pattern, and make the successive changes of eccentricity by turning the screw of the slide-rest forward. In the first

case we increase, and in the second case we diminish, the eccentricity between each cut. Whichever plan is adopted should be adhered to throughout the pattern, so as to avoid loss of time.

22. The beauty of these patterns depends on the work being done with considerable accuracy; the workman must therefore make the adjustment of all at centre most carefully, he must avoid loss of time, and must not forget the existence of the central reading. Although he *may* add the central reading in his head to each successive tabular value of the eccentricity while he is cutting the pattern, he is most strongly advised not to do so, but to write down the apparent eccentricities before he begins his work. He is also advised to adopt the method suggested at Chapter III., Part II., Article 13 (*ante*, p. 92).

23. It is worth remembering that each circle produced by cutting at any pair of columns given in the Tables contains forty-eight cuts, situated symmetrically with respect to the centre of that circle. The workman will readily apply the rules (*ante*, p. 35) for putting the cuts in contact. He must remember, however, that the angular distance from one cut to another, measured with

respect to the circle in which they both lie, is double the angular distance through which the work has been turned. For instance, the cuts made at 24 and 48 in the same circle lie at opposite extremities of a diameter, so that the angular distance between those cuts with respect to that circle is 180°, though, in passing from one to the other, the work has to be turned through 90° only.

Let it be required, for example, to place six circles in contact on the circle described by cutting in the pair of Columns I., TABLE I., when $d = 100$. Now d, the diameter of the circle, is twice the eccentricity of the cut at Z as measured, not from the centre of the work, but from the centre of the circle, so that we must take $\frac{d}{2}$ as the value of e in the "TABLES FOR CIRCLES IN CONTACT." From those Tables we have: taking

$$e = \frac{d}{2} = \frac{100}{2} = 50;$$

$$\text{and } n = 6$$

$$r = 25.$$

Instead, however, of making cuts at every $(\frac{96}{6})^{th}$ or 16^{th} division on the plate, as we should if the circle to be described had no eccentricity, we must make them at every 8th division, at

96, 8, 16, 24, 32, 40, for instance; or at
5, 13, 21, 29, 37, 45.

It must be remembered in designing patterns, that if either TABLE I. or TABLE II. is used, each cut at N. occurs in two adjacent circles; and therefore, if possible, the cuts in the figure should be arranged so as to comprise N.

Should the TABLE III. be used, similar remarks apply to R; in the same Table cuts J occur in alternate circles, so that similar remarks apply to J, if the pattern is to comprise alternate circles only.

The reader is advised to draw with compasses four circles with equal diameters, meeting in a point and having their central diameters at right angles to each other, and to write the principal cuts of TABLE I. and TABLE II. in their proper places. He may afterwards proceed in a similar manner with respect to TABLE III., but in this case any two adjacent central diameters must contain an angle of 60°.

Example.—Let the direction be

$r = 10\ d = 50$

A to N, Table I.

$r = 10\ d = 75$

N to Z, Table II.

These directions become, when translated into ordinary language :

"With radius 10 and eccentricity 0, cut on the scale of 96 at

96, 48, 24, 72, 48, 96, 72, 24."

It is unnecessary to add, that the cut need be made at one of these numbers only, as the cut occupies the same position at whichever number it is made.

"With eccentricity $3\frac{1}{4}$, cut at

1 47 25 71 49 95 73 24 ;

with eccentricity $6\frac{1}{2}$, cut at

2 46 26 70 50 94 74 23 ;

with eccentricity $9\frac{3}{4}$, cut at

3 45 27 96 51 93 75 22 ;

with eccentricity 13, cut at

4 44 28 68 52 92 76 21 ;

with eccentricity 16, cut at

5 43 29 67 59 91 77 20;

with eccentricity 19, cut at

6 42 30 66 54 90 78 19;

with eccentricity 22, cut at

7 41 31 65 55 89 79 18;

with eccentricity 25, cut at

8 40 32 64 56 88 80 17;

with eccentricity $27\frac{3}{4}$, cut at

9 39 33 63 57 87 81 16;

with eccentricity $30\frac{1}{2}$, cut at

10 38 34 62 58 86 82 14;

with eccentricity 33, cut at

11 37 35 61 59 85 83 13;

with eccentricity $35\frac{1}{4}$, cut at

12 36 36 60 60 84 84 12."

We have now completed the first part of the pattern; it consists of four half-circles, cutting each other in the middle points in the centre of the work at right angles to each other.

The remaining part of the pattern consists of the halves, farthest from the centre of the work, of four circles whose central diameters are at right angles to each other, and which occupy an intermediate position to the circles part of which have already been cut.

We continue the pattern as follows :—

"With eccentricity 53, cut at

24 48 48 72 72 96 96 24;

with eccentricity 56½, cut at

25 47 49 71 73 95 1 23;

with eccentricity 59½, cut at

26 46 50 70 74 94 2 22;

with eccentricity 62¼, cut at

27 45 51 69 75 93 3 21;

with eccentricity 64¾, cut at

28 44 52 68 76 92 4 20;

with eccentricity 67¼, cut at

29 43 53 67 77 91 5 19;

with eccentricity 69¼, cut at

30 42 54 66 78 90 6 18;

with eccentricity 71, cut at

31 41 55 65 79 89 7 17;

with eccentricity 72¼, cut at

32 40 56 64 80 88 8 16;

	with eccentricity $73\frac{1}{2}$, cut at						
33	39	57	63	81	87	9	15 ;
	with eccentricity $74\frac{1}{4}$, cut at						
34	38	58	62	82	86	10	14 ;
	with eccentricity $74\frac{1}{2}$, cut at						
35	37	59	61	83	85	11	13 ;
	with eccentricity 75, cut at						
36	36	60	60	84	84	12	12."

24. The mathematician will observe that the circle whose equation is

$$e = d \sin \theta,$$

and also the cardioid whose equation is

$$e = a\,(1 + \cos \theta)$$

is a particular case of the curves treated of in the second part of this Chapter.

The author has given settings for cutting the cardioid at the end of the settings for circular figures.

PART IV.

STRAIGHT LINES.

25. THE figures produced by the methods contained in this part of Chapter III. are intended to be cut with the slide-rest in the transverse position, and consist entirely of figures in which the cuts are arranged in straight lines not passing through the centre of the work. No doubt such figures have occasionally been cut by raising the slide-rest above the centre of the work; but having regard to the great difficulty of raising it through a definite distance, it is probable that this has been done but rarely, and that, as a general rule, figures of the nature here described have been produced by means of the eccentric chuck used in conjunction with the cutting-frame and the slide-rest movement.

26. In the directions for cutting the patterns the direction "$r = 25$" means that the cuts are to be made with a radius 25, or that the slide of the eccentric cutting-

frame is to be thrown out two and a half turns (*ante*, p. 17).

27. There are two Tables for straight lines. The first, headed "TABLE FOR STRAIGHT LINES NO. I.," is calculated for the Division-plate, Scale 96; the second, headed "TABLE FOR STRAIGHT LINES NO. II." is calculated for the Division-plate, Scale 120. Each Table is divided into vertical lines, headed by the numbers 100, 80, &c., and into horizontal lines, denoted by the letters *A*, *B*., &c.

The numbers in any vertical column are the eccentricities for making cuts at the successive numbers on the division-plate for the purpose of cutting a straight line whose distance from the centre of the work is the number at the head of that column.

Thus, if we wish to cut a straight line whose distance from the centre of the work equals 50, we should (supposing that we cut from TABLE I.) make a cut at any number (96 suppose) on the Scale of 96, with eccentricity 50; we should then cut at 95 and 1 with eccentricity 50, then at 94 and 2 with eccentricity 50·1, and so on.

If it were possible to make cuts at all the numbers in any column, the straight line would reach to infinity in both directions. In practice the cuts become too far apart for most patterns after Q in TABLE I. and after U in TABLE II.

28. In the directions for cutting a pattern, the particular column in the Table that is to be taken is indicated by the direction "$p = 30$," or whatever the number may be. Thus the direction

$$p = 30$$

means, that the eccentricities for the successive cuts are to be taken from the column headed 30 in TABLE I., or TABLE II. as the case may be; it also indicates that the nearest distance from the straight line intended to be described to the centre of the work equals 30.

29. In the directions for cutting a pattern, the letter A, standing alone or separated from the following letters by a comma, means, "Cut at any number that you please, with the eccentricity given by the Table opposite to *A*;" B means, "Cut at the two numbers adjacent to A, with the eccentricity given in the Table opposite to *B*;" C means, "Cut at the two numbers respectively greater and

less by 2 than A, with the eccentricity given in the Table opposite to *C*," and so on.

It will be observed that the numbers in any one of the columns of the "TABLES FOR LOOPED FIGURES," headed "DIVISION-PLATE SCALE" follow the law above mentioned; the numbers opposite B are the numbers adjacent to each of the numbers opposite A, while each of the numbers opposite C differ by two from the numbers opposite A, the one being greater and the other less than them. It follows, therefore, that if cuts be made at the numbers in any one of these columns, with the successive eccentricities *A B*, &c., a straight line will be produced; and if cuts be made at the numbers in all these columns, we shall have as many straight lines as there are columns, each straight line being at the same distance from the centre of the work as the others, and every two adjacent straight lines containing the same angle. In other words, the straight lines would, or if the proper number of cuts in each were made they would, produce the regular polygons.

Whenever the direction is, "Cut from the TABLE OF — LOOPED FIGURES," any letter, as C, standing alone or separated from the following letter by a comma only, means, Make cuts with the same radius at all the numbers in the horizontal line C in the given Table for looped figures, with the eccentricity in the horizontal line *C* and the given vertical column of the TABLE FOR STRAIGHT LINES, taking Table I. or II. according as the Scale of the given TABLE FOR LOOPED FIGURES is 96 or 120.

For example, let the directions be

$$r = 10 \quad p = 50,$$

cut from TABLE FOR 5-LOOPED FIGURES,

C.

We should cut at

4, 8, 28, 32, 52, 56, 76, 80, 100, 104,

with eccentricity 50¼, using the TABLE No. II., because the TABLE FOR 5-LOOPED FIGURES is adapted to the Scale of 120.

When the direction is given to cut from a particular Table for looped figures, any two or more of the letters A B, &c., occurring together, not separated by a comma or full stop, means, "Make cuts with the same radius at all the numbers that occur in the same horizontal lines as such letters, with the eccentricity corresponding to the first of them."

Thus the direction

"$r = 10 \quad p = 80$,

cut from TABLE FOR 4-LOOPED FIGURES

DK,"

means, "Make cuts with the same radius (10) at the numbers

3, 9, 27, 33, 51, 57, 75, 81,
93, 15, 21, 39, 45, 63, 69, 87,

with eccentricity $81\frac{1}{2}$," which corresponds to D, while if the direction had been KD, we should have made the cuts with the eccentricity $96\frac{1}{4}$, which corresponds to K; where we use TABLE FOR STRAIGHT LINES, No. I., as the TABLE FOR 4-LOOPED FIGURES is calculated for the Scale of 96.

30. A comma denotes a change of eccentricity. As a general rule, it is convenient to make the first cut at the inside of the pattern, *i. e.*, as near as possible to the centre of the work, and to make the successive changes of eccentricity by turning the screw of the slide-rest

forwards, thus increasing the eccentricity between each cut.

31. The beauty of these patterns depends *entirely* on the work being done with accuracy; the workman must therefore make the adjustment for all at centre most carefully; he must avoid loss of time, and must not forget the existence of the central reading. Although he may add the central reading in his head to each successive tabular value of the eccentricity, while he is cutting the pattern, he is most strongly advised not to do so, but to write down the apparent eccentricities before he begins to work. He is also advised to adopt the method suggested in Chapter III., Part II., Article 13 (*ante*, p. 92).

Example.—Let the directions be

$$r = 10 \quad p = 50$$

AR, BQ, CP, DO, EN, FM, GL, HK, J.

cut from Table for 3-Looped Figures.

These directions become, when translated into ordinary language:

"With radius 10 and eccentricity 50, cut, on the Scale of 96, at

8,		40,		72,	
	24,		56,		88;

with eccentricity 50, cut at

7, 9,	39, 41,	71, 73,
89, 23,	25, 55,	57, 87;

with eccentricity $50\frac{1}{2}$, cut at

6, 10,	38, 42,	70, 74,
90, 22,	26, 54,	58, 86;

with eccentricity 51, cut at

5, 11,	37, 43,	69, 75,
91, 21,	27, 53,	59, 85;

with eccentricity $51\frac{3}{4}$, cut at

4, 12,	36, 44,	68, 76,
92, 20,	28, 52,	60, 84;

with eccentricity $52\frac{3}{4}$, cut at

3, 13,	35, 45,	67, 77,
93, 19,	29, 51,	61, 83;

with eccentricity 54, cut at

2, 14,	34, 46,	66, 78,
94, 18,	30, 50,	62, 82;

with eccentricity $55\frac{3}{4}$, cut at

1, 15,	33, 47,	65, 79,
95, 17,	31, 49,	63, 81;

with eccentricity $57\frac{3}{4}$, cut at

96, 16,	32, 48,	64, 80."

32. Some very curious figures can be produced by taking the successive changes of eccentricity from the Table, but making the eccentricity for the first cut greater or less than that given in the Table. If the first cut is made at the part of the figure nearest to the centre of the work, with an eccentricity greater than that given in the Tables, a convex curve will be produced, while if it be made with an eccentricity less than that given in the Tables, a concave curve will be produced; where I use the words convex and concave, supposing that we look towards the centre of the work. See Ash, "Double Counting," Plate 60.

PART V.

THE ELLIPSE.

33. The figures produced by the methods contained in this part of Chapter III. are intended to be cut with the slide-rest in the transverse position, and consist entirely of figures in which the cuts are arranged in ellipses whose centre coincides with the centre of the work. The author is not aware that patterns in the form of an ellipse have hitherto been produced without the employment of either the oval chuck or the eccentric chuck.

34. It will be observed that one ellipse differs from another not only in absolute magnitude, but also in the ratio that its axes bear to each other; the author has therefore given four Tables for the ellipse; each Table containing the settings for ellipses whose axes bear a constant ratio to each other, or in other words, which are similar to each other; while the axes of an ellipse

cut from one Table bear a different ratio to each other, to that which the axes of an ellipse cut from another Table bear to each other; it follows that if the radius of the cuts is small, different ellipses of different sizes can be cut from the same Table without intersecting each other.

35. Each of the "Tables for the Ellipse," *post*, pp. 206, *et seq.*, is calculated for the Division-plate, Scale 96. It is arranged in horizontal lines, headed by the letters *A*, *B*, *C*, &c., and vertical lines, headed by the numbers 100, 80, &c. If we make cuts at consecutive numbers on the Division-plate, Scale 96, with the successive eccentricities given in any column, we shall produce a quarter of an ellipse, whose semi-axis major is the number at the head of the column, and whose semi-axis minor is the last number in the column.

36. In the directions for cutting a pattern, the direction

$$\text{“}r = 10\text{”}$$

means that the cuts are to be made with a radius 10, or that the slide of the eccentric cutting-frame is to be thrown out one complete turn (*ante*, p. 17).

37. The particular Table to be employed is denoted by the direction

"TABLE I.," or "TABLE II."

The particular column to be employed is denoted by the direction

"$a = 100$,"

or 80, or whatever the number may be, where 100 or 80 is the number that heads that column.

In the directions for cutting a pattern any one of the letters A, B, C, &c. standing alone, or separated from the following letters by a comma (,), means that the workman is to "Make cuts with the same radius at all the numbers on the division-plate that occur in the TABLE FOR 2-LOOPED FIGURES AND THEIR DERIVATIVES, No. I., in the same horizontal line as that letter, with the eccentricity found in the given vertical column of the given Table for the ellipse in the horizontal line headed by the corresponding italic letter."

Example.—Let the directions be

TABLE I. $a = 50$ $r = 10$

C.

This means, "Make cuts with radius 10 at the numbers

10 14 58 62,

with the eccentricity $49\frac{3}{4}$."

38. Any two or more of the letters A B C, &c. occurring together, not separated by a comma or full stop, mean, "Make cuts with the same radius at all the numbers on the division-plate that occur in the TABLE FOR 2-LOOPED FIGURES AND THEIR DERIVATIVES, No. I., in the same horizontal lines as those letters, with the eccentricity found in the given column of the given Table of the ellipse, in the horizontal line headed by the first of such letters in italics."

Example.—Let the direction be

TABLE I. $a = 50$ $r = 10$
DK.

This means, "Make cuts with radius 10 at the numbers

9 15 57 63
3 21 51 69,

with eccentricity $48\frac{1}{2}$."

While if the direction had been KD, we should have had to make the same cuts with the eccentricity 46¼.

39. It is convenient to remember that the cuts A are at the extremities of the axis major of the ellipse, that the cuts Z are at the extremities of the minor axis, and that the cuts N are the middle cuts of each elliptic quadrant, or as Captain Ash calls them, are the shoulders of the ellipse.

Some curious patterns may be produced by increasing or diminishing the eccentricity of every cut by the same quantity. Thus, if the eccentricity of every cut be diminished by the eccentricity of the axis major, we shall produce a 2-looped figure.

I have considered it unnecessary to work out an example at length.

40. The Table for Waved Ellipses corresponds in general arrangement with the other Tables. If cuts be made at the numbers corresponding to each of the letters A, B, &c., in the Table for 2-Looped Figures with the corresponding eccentricities taken from any one of the vertical columns in the Table for Waved Ellipses, the figure will be produced.

This Table has been formed in the manner following:

Suppose that in the directions for cutting 2-looped figures we had given

$$e = 100\ b = 50 \text{ from } A,$$
AGNTZ, BFHMOSUY, CEJLPRVX, DKQW,

we should cut at the letters AGNTZ with eccentricity 100, and should diminish the eccentricity by $\frac{1}{2}$ for the cuts BFHMOSUY, by $\frac{1}{2} + 1\frac{1}{4} = 1\frac{3}{4}$ for the cuts CEJLPRVX, and by $\frac{1}{2} + 1\frac{1}{4} + 2 = 3\frac{3}{4}$ for the cuts DKQW, thus producing a figure with convex waves; and if we diminish the eccentricities for the corresponding cuts of the ellipse by the same numbers, we shall produce an ellipse with convex waves. The eccentricities in columns I. and III. of the TABLE FOR WAVED ELLIPSES are respectively calculated in this manner from the first and second Tables for the ellipse.

Similarly had the direction been

$$e = 100 \quad b = 50 \text{ from } W,$$
AGNTZ, BFHMOSUY, CEJLPRVX, DKQW,

we should have a figure with concave waves; the total changes of eccentricity for the successive cuts being 2, $3\frac{1}{4}$, $3\frac{3}{4}$.

The eccentricities in columns II. and IV. of the TABLE FOR WAVED ELLIPSES are calculated by diminishing the corresponding eccentricities for the ellipses given in Tables I. and II. by the numbers given in the last paragraph.

CHAPTER V.

ENVELOPES.

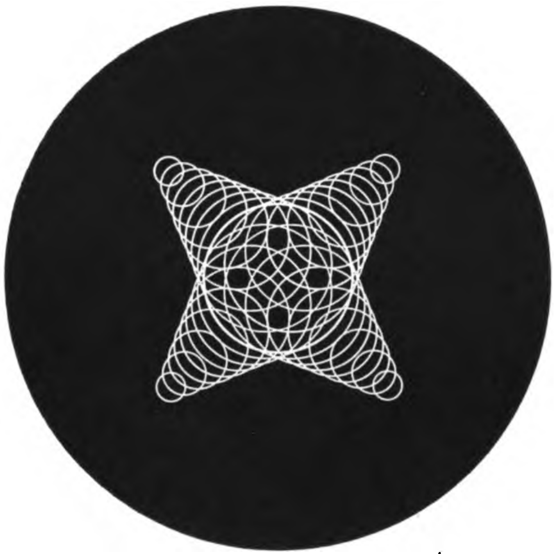
1. When we cut any one of the patterns described in the preceding Chapters, with the exception of the shell, the radius remains constant throughout the pattern. We now pass to the consideration of those patterns in which the radius is variable. Such patterns have hitherto excited very little notice; in fact, they appear, with the exception of the shell and the star, to be unknown.

To cut a Star. Cut a circle of any radius that you please with no eccentricity. Make all the following cuts at as many numbers as the star is to have rays, equidistant from each other on the division-plate. For example, if the star is to have four rays, make all the following cuts at

96, 24, 48, 72.

After the first cut is made, diminish the radius by any

Plate 65.



quantity that you like, and increase the eccentricity by any larger quantity; cut at

96, 24, 48, 72;

repeat the process until the star is complete, the quantities by which the radius is diminished and the eccentricity increased between each cut being constant.

Example.—Cut at

96, 24, 48, 72,

with the following corresponding values of r and e.

r = 26	24	22	20	18	16	14	12	10	8	6
e = 0	10	20	30	40	50	60	70	80	90	100

The reader will be able to devise stars of different forms for himself; but, in the author's opinion, they are not worth the trouble of cutting.

2. Place the slide-rest in the transverse position, surface a piece of wood, draw with a pencil any curve that you like on it. As pointed out (*ante*, p. 67), we can cut dots with the eccentric cutting-frame along the curve, one dot corresponding to each number on the division-plate,

and to some particular eccentricity, *e*. Now, suppose that when the index is in 96 we diminish the eccentricity, and throw out the slide of the eccentric cutting-frame by the same amount, and cut a circle; this circle will pass through the dot cut at 96. Now move the index to 1; we shall find that (unless our pencil-line is a circle whose eccentricity is zero) if we cut a circle with the same radius, it will not pass through the dot cut at 1, but that, if we alter the radius, or, if we prefer it, alter both the radius and the eccentricity, we can make it do so; and, proceeding in this manner, we shall be able to make a number of circles, each with a different radius, and each passing through a dot in the original cut. If we make the circles small enough, the outline of the pattern produced will be the pencil-curve. The reason for saying "if we make the circles small enough" is the following: each circle will meet the pencil-line at the point corresponding to the dot cut at the same number as itself; but if the radius be too large, it may also cut the pencil-line at some other point, thus spoiling the pattern.

The author proposes to call patterns cut with a variable radius by the name "Envelopes" for a reason which the mathematician will perceive. The number of envelopes that can be cut is only limited by the skill of the workman.

He considers it convenient to give an explanation addressed to the mathematician of the settings for each example that he gives of this method. He has, however, given the directions for cutting the patterns in a form adopted for the un-mathematical reader.

The workman must not judge of the beauty of these patterns by the figures that are printed, as their beauty depends in great measure on the play of light from their surface, which sometimes assumes curious and unexpected forms.

It is perhaps necessary to warn the workman that it is extremely difficult to cut an envelope properly; the slightest error in the position of any one of the cuts may ruin the appearance of the whole pattern.

3. Let it be required to make the cuts touch two given straight lines which meet at a point whose eccentricity is E, which contain an angle 2α, and which are arranged symmetrically with respect to the centre of the work; then we shall have

$$r = (E - e) \sin \alpha$$

or

$$e = E - r \operatorname{cosec} \alpha.$$

We shall be able, therefore, to take either the successive values of r or of e at our pleasure.

Let us apply this formula to the production of the regular polygons up to and including the hexagon.

The values of the sin a are given in the following tables.

Number of Sides of the Polygon	3	4	5	6
$2a$	60°	90°	108°	120°
a	30°	45°	54°	60°
Sin a	·5	·707	·809	·866

To save trouble to the reader, the author has given the values of the radius and eccentricity, calculated according to the Table given above, for every cut in each of the regular polygons as far as the hexagon. The calculations have been made on the supposition that the eccentricity of the first cut is 100, and that the radius is zero. Inspection of the figures will show that the patterns would have looked better if the first cut had been omitted. Should it be desired to cut the patterns on a different scale, this can easily be done by changing the eccentricity and radius of every cut in the same ratio.

4. Let the cuts lie on a circle whose centre is the centre of the work, and let them touch a given straight line.

Let E be the eccentricity of the point of the line nearest to the centre of the work, and let R be the radius of the given circle.

We shall have

$$r = E - R \cos \theta;$$

where θ is measured from the perpendicular let fall on the given straight line from the centre of the work.

As the figures produced in this manner have rounded corners and have no particular beauty, the author considers it unnecessary to make the calculations.

5. Let the cuts be arranged on a circle whose circumference passes through the centre of the work, and let them touch a tangent of that circle which is parallel to the central diameter.

Let θ be the angle contained between the central diameter and the radius of the given circle which passes through the centre of any particular cut, then we shall have

$$r = \frac{d}{2}(1 - \sin\theta);$$

and if ϕ be the angle that the division-plate has to be moved through, in passing from the cut at the centre of the work to the cut in question we shall have

$$\frac{\phi}{2} = \theta;$$

we find therefore that we can make the cuts required by cutting them at the numbers given in any one of the columns in the TABLE FOR CIRCLES No. I., II., or III., with the corresponding eccentricity, and with a radius given by the formula

$$r = \frac{d}{2}\left(1 - \sin\frac{\phi}{2}\right).$$

It will be observed that if the circles be cut at the numbers in both the columns in the same pair, we shall make them touch both the tangents to the given circle which are parallel to the central diameter.

6. Let the cuts be arranged in a circle, and let the

radius vary as the sine of the angle through which the division-plate is moved.

It will be found that the outline of the pattern is the outer half of the cardioid.

7. Take any figure produced by dual counting: change the value of r in any manner that you please between each cut. If e be the eccentricity for any cut according to the rules for dual counting, and e_1 and r_1 be respectively the eccentricity with which the cut is actually made, take e_1 such that

$$e = e_1 + r_1$$

or, in other words, let the sum of the radius and eccentricity with which any cut is made equal the eccentricity according to the rules of dual counting for the corresponding cut.

Example.—Cut a square according to the method given *post*, p. 172.

$p = 40$, TABLE I.

Make the cuts with the following values of r and e_1:—

	A	B	C	D	E	F	G	H	J	K	L	M	N
r	16	15	14	13	12	11	10	9	8	7	6	5	4
e	24	25	26¼	27¾	29¼	31¼	33¼	35¾	38¼	41	44¼	48¼	52½

It will be found that the sum of the values of r and e_1 for any cut equals the tabular value of e.

Plate 69.

Plate 70.

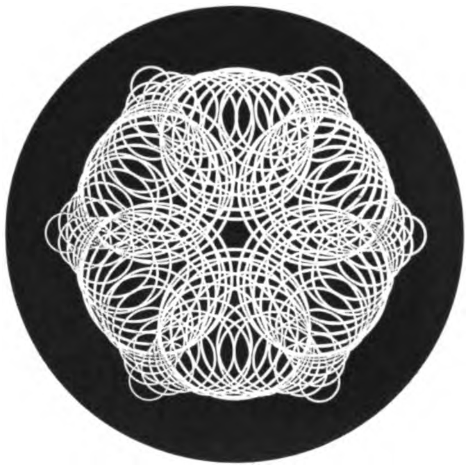


Plate 6.

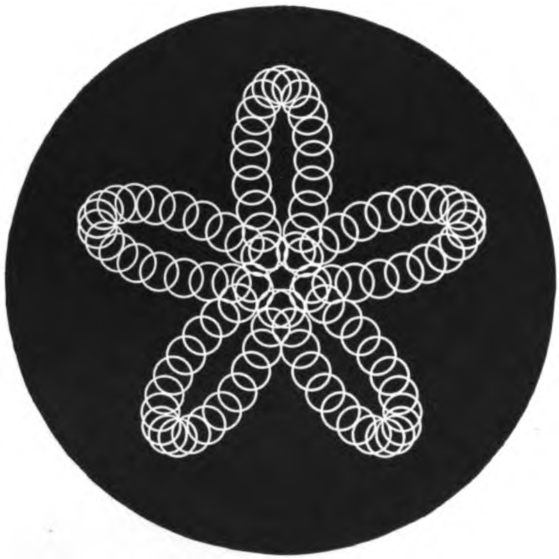
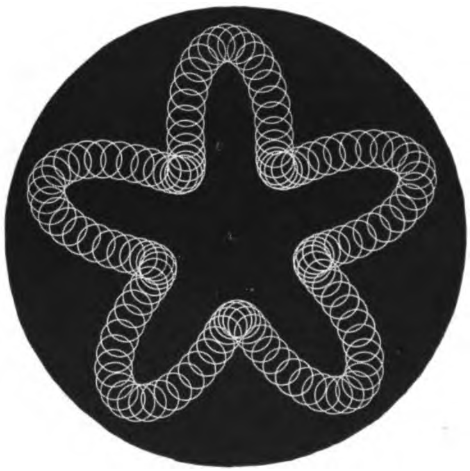


Plate 8.



SETTINGS FOR DUAL COUNTING.

PART I.

SETTINGS FOR 5-LOOPED FIGURES AND THEIR DERIVATIVES.

PATTERN 1.

$r = 10$ $e = 100$ $b = 30$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

Here as $2\ b$ is less than 100, the curve does not reach the centre of the work.

PATTERN 2.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

Here as $2\ b = e$, the curve exactly reaches the centre of the work: the cuts M, or some of them, may be omitted, so as to avoid crowding the centre of the work.

PATTERN 3.

$r = 10 \qquad e = 100 \qquad b = 100$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

Attend to the rule as to negative values of eccentricity, *ante* p. 77.

As 2 b is greater than e, the curve passes beyond the centre.

PATTERN 4.

$r = 10 \qquad e = 100 \qquad b = 100$ from A.

AN, BM, CL, DK, EJ, FH, G.

This is derived from Pattern 3, by repeating the first half of the pattern and displacing it angularly.

PATTERN 5.

$r = 10 \qquad e = 100 \qquad b = 50$ from A.

AN, BM, CL, DK, EJ, FH, G.

This is derived from Pattern 2 in a manner similar to that in which Pattern 4 is derived from Pattern 3.

PATTERN 6.

$r = 10 \qquad e = 100 \qquad b = 50$ from G.

AN, BM, CL, DK, EJ, FH, G.

This is the second half of Pattern 2, cut with greater eccentricity.

Plate 5.

PLATE 7.

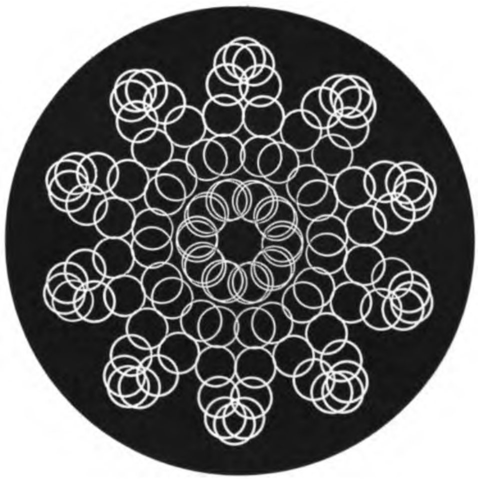


Plate 9.

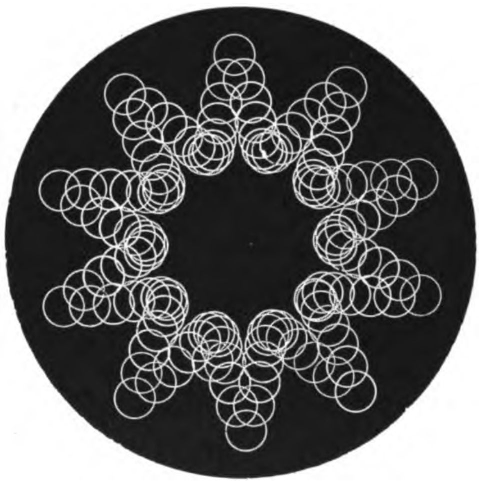


Plate 10.

PLATE 11.

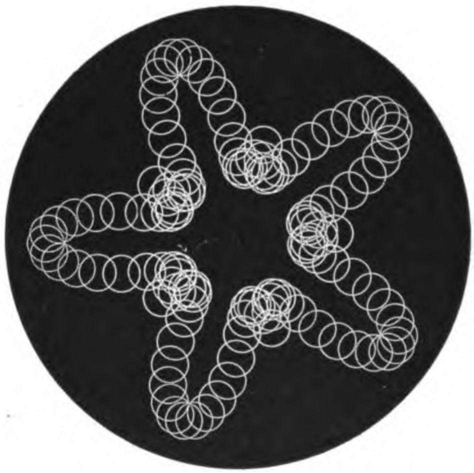
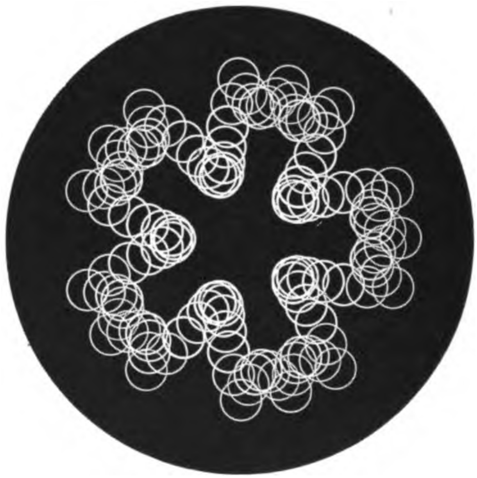


Plate 12.



PATTERN 7.

$r = 8$ $e = 100$ $b = 50$ from A.

A, B, C, [DK,] E, F.

$b = 50$ from A.

DK, JL, HM, N.

This Pattern should be cut very deep.

PATTERN 8.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, [GN.]

$b = 50$ from K.

GN, HM, JL, K.

This would perhaps be improved by making $e = 85$, and omitting JL.

PATTERN 9.

$r = 10$ $e = 100$ $b = 50$ from A.

K, JL, HM, [GN].

$b = 50$ from G.

GN, F, E, D, B, C, A.

PATTERN 10.

$r = 10$ $e = 100$ $b = 50$ from G.

[GA], H, J, K, L, M, N.

$b = 50$ from K.

GA, FB, EC, D.

PATTERN 11.

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

DK, CEJL, BFHM, AGN.

for centre, repeat with $e = 40$.

The Central Pattern must be cut with very great care, and should be merely scratched.

PATTERN 12.

$r = 10$ $e = 100$ $b = 50$ (or 30) from K.

AGN, BFHM, CEJL, DK.

for centre, repeat with $e = 40$.

See note as to last Pattern, as to cutting Central Pattern.

PATTERN 13.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, [GN].

$b = 50$ from K.

GN, HM, JL, K, cutting the numbers in the even columns, II., IV., &c., only.

$e = 75$ $b = 50$ from A.

K, JL, HM, cutting the numbers in the odd columns, I., III., &c., only.

In the figure F is omitted.

Plate 13.

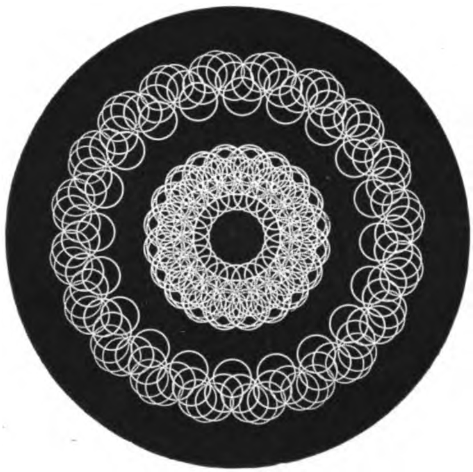


Plate 14.

Plate 15.

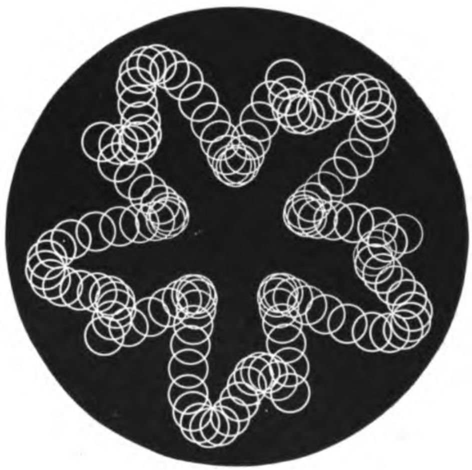


Plate 16.

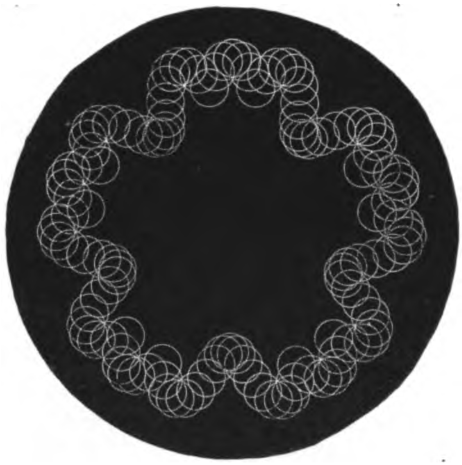


Plate 17.

PATTERN 14.

$r = 10 \qquad e = 100 \qquad b = 50$ from A.

K, JL, HM, [GN], cutting at the numbers in the even columns only (except GN).

$b = 50$ from G.

GN, F, E, D, C, B, A.

$b = 50$ from K.

GN, HM, JL, K, cutting at the numbers in the odd columns only.

PATTERN 15.

$r = 10 \qquad e = 100 \qquad b = 50$ (or 30) from A.

DK, CEJL, BFHM, (AGN);

cutting H, J, K, L, M in even columns only.

$b = 50$ or 30 from K.

AGN, HM, JL, K, cutting at odd columns only.

This may also be cut as a Central Pattern with $e = 50$.

PATTERN 16.

$r = 10 \qquad e = 100 \qquad b = 50$ (or 30) from A.

D, CE, BF, [AGN].

$b = 50$ (or 30) from K.

AGN, HM, JL, K.

If the Pattern be cut with $b = 30$ there is room for a figure in the centre. In which case the Pattern may be repeated with $e = 50$.

PATTERN 17.

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

D, CE, BF, [AGN];

cutting at the numbers in the odd columns only of B, C, D, E, F.

$b = 50$ (or 30) from K.

AGN, BFHM, CEJL, DK;

cutting at the numbers in the even columns only of B, C, D, E, F.

If the Pattern be cut with $b = 30$ there is room for a Pattern in the centre. In which case the Pattern may be repeated with $e = 50$.

PATTERN 18

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

DK, CEJL, BFHM, [AGN], cutting at odd columns only (except for AGN).

$b = 50$ (or 30) from K.

AGN, BFHM, CEJL, DK, cutting at even columns only.

If the Pattern be cut with $b = 30$ there is room for a Pattern in the centre. In which case the Pattern may be repeated with $e = 50$.

PATTERN 19.

$r = 10$ $e = 100$ $b = 100$ from A.

AN, BM, CL, [DK].

$b = 100$ from K.

DK, EJ, FH, G.

(or omit FH, G).

PLATE 18.

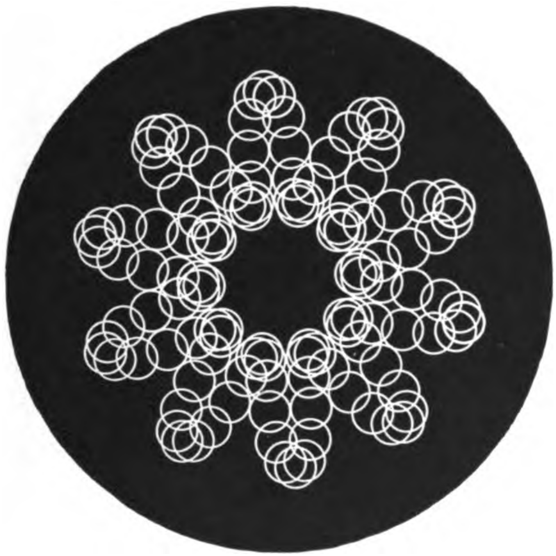


PLATE 19.

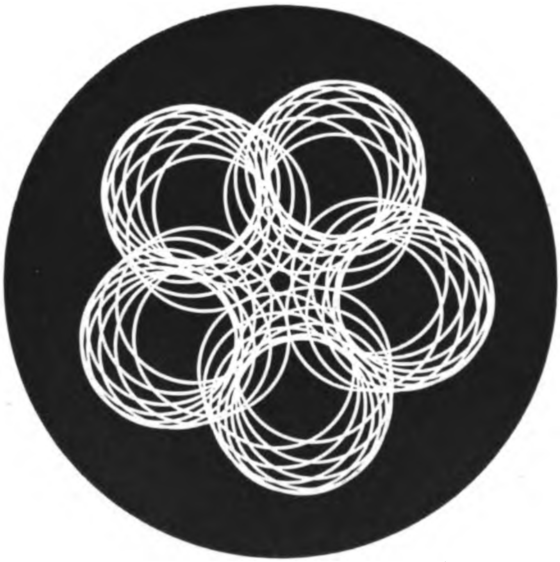


Plate 20.

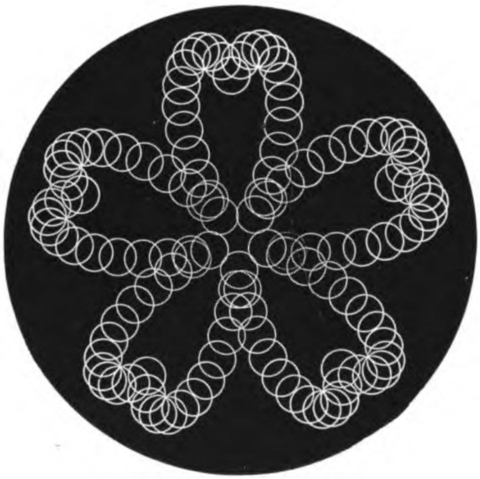


PLATE 21.

PATTERN 20.

$r = 40$ $e = 70$ $b = 50$ from A.

A, B, C, D, E.

PATTERN 21.

$r = 40$ $e = 70$ $b = 25$ from A.

A, B, C, D, E, F, G.

PATTERN 22.

$r = 10$ $e = 100$ $b = 50$ from A.

D, EC, FB, GA, H, J, K, L, M, N.

This might be varied by cutting from GA with $b = 50$ from G.

PATTERN 23.

$r = 10$ $e = 50$ $b = 50$ from A.

A, B, C, D, E, F, G.

PATTERN 24.

$r = 6$ $e = 50$ $b = 25$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

Some of the last cuts to be blind, to avoid confusion.

PATTERN 25.

$r = 10$ $e =$ from 100 to 50 $b = 50$ from A.

[DN], CE, BF, AG.

$b = 30$ from A, giving to Δe half its tabular value.

DN, M, L, K, J, H.

PATTERN 26.

$r = 10$ $e = 100$ to 50 $b = 50$ from K.

[AG], BF, CE, D.

$b = 30$ from G, giving Δe half its tabular value.

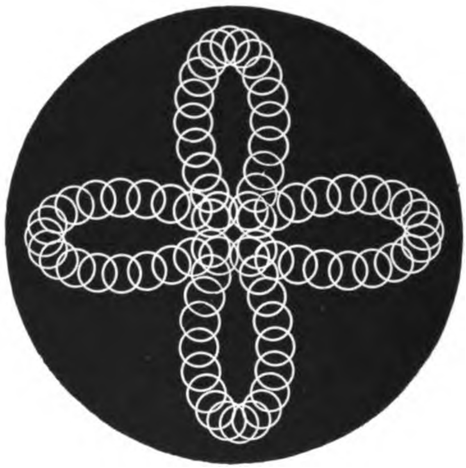
AG, H, J, K, L, M, N.

PATTERN 27.

$r = 12$ $e = 80$ $b = 50$ from A.

AGN, BFHM, CEJL, DK.

PLATE 22.



PART II.

SETTINGS FOR 4-LOOPED FIGURES AND THEIR DERIVATIVES.

PATTERN 1.

$r = 10 \quad e = 100 \quad b = 30$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

PATTERN 2.

$r = 10 \quad e = 100 \quad b = 25$ from A.

AN, BM, CL, DK, EJ, FH, G.

PATTERN 3.

$r = 10 \quad e = 100 \quad b = 50$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

Some of the last cuts at L, M, N, should be omitted according to the taste of the workman, so as to avoid crowding at the centre.

PATTERN 4.

$r = 10$ $e = 100$ $b = 100$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

Attend to rule as to negative values of eccentricity, or cut

AN, BM, CL, DK, EJ, FH, G.

PATTERN 5.

$r = 10$ $e = 100$ $b = 50$ from A.

AN, BM, CL, DK, EJ, FH, G.

PATTERN 6.

$r = 15$ $e = 100$ $b = 50$ from G

AN, BM, CL, DK, EJ, FH, G.

PATTERN 7.

$r = 10$ $e = 70$ $b = 30$ from A

A, B, C, D, [E K], F.

$b = 30$ from A.

EK, JL, HM, GN.

PATTERN 8.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, [G N];

$b = 50$ from K.

GN, HM, JL, K.

Plate 23.

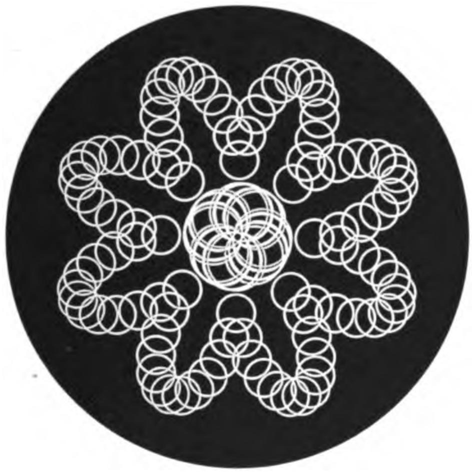


PLATE 24.

PLATE 25.

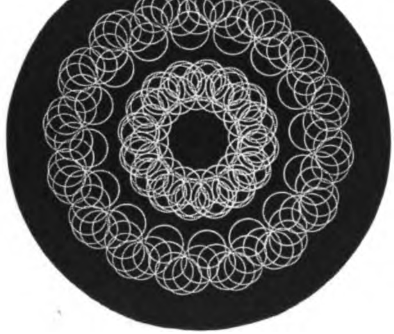
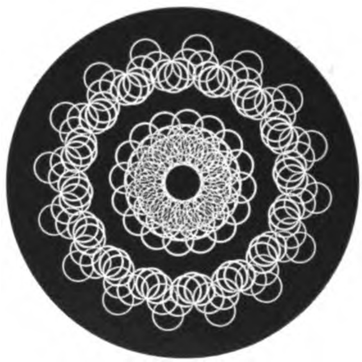


Plate 26.



PATTERN 9.

$r = 10$ $e = 100$ $b = 50$ from A.

K, JL, HM, [GN].

$b = 50$ from G.

G, N, F, E, D, B, C, A.

PATTERN 10.

$r = 10$ $e = 100$ $b = 50$ from G

[G A], H, J, K, L, M, N.

$b = 50$ from A.

GA, FB, EC, D.

PATTERN 11.

$r = 8$ $e = 70$ $b = 50$ from A.

AGN, BFHM, CEJL, DK.

for centre $e = 30$ $b = 50$ from A

AGN, BFHM, DK.

PATTERN 12.

$r = 8$ $e = 70$ $b = 50$ from K.

AGN, BFHM, CEJL, DK.

for centre $e = 30$ $b = 50$ from K.

AGN, CEJK, DK.

PATTERN 13.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, [GN].

$b = 50$ from K.

GN, HM, JL, K, cutting at even columns only.

$e = 75$ $b = 50$ from A.

K, JL, HM, cutting at odd columns only.

PATTERN 14.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, [GN].

$b = 50$ from K.

GN, HM, JL, K, cutting at columns 3, 4, 7, 8, only.

$e = 75\frac{1}{2}$ $b = 50$ from A.

K, JL, HM, cutting at columns 1, 2, 5, 6, only.

PATTERN 15.

$r = 10$ $e = 100$ $b = 50$ from A.

K, JL, HM, [GN], cutting except GN, at columns 3, 4, 7, 8, only.

$b = 50$ from K.

[GN], HM, JL, K : cutting at columns 1, 2, 5, 6, only.

$b = 50$ from G.

GN, F, E, D, C, B, A.

Plate 27.

PATTERN 16.

$r = 10$ $e = 100$ $b = 50$ from A.

K, JL, HM, [GN], cutting, except GN, at odd columns only.

$b = 50$ from G.

[GN], F, E, D, C, B, A.

$b = 50$ from K.

GN, HM, JL, K, cutting at even columns only.

PATTERN 17.

$r = 10$ $e = 100$ to 40 $b = 50$ from A.

DK, CEJL, BFHM, [AGN];

cutting HJKLM in even columns only.

$b = 50$ from K.

AGN, HM, JL, K;

cutting in even columns only.

PATTERN 18.

$r = 10$ $e = 100$ or 70 $b = 50$ (or 30) from A.

D, CE, BF, [AGN].

$b = 50$ or 30 from K.

AGN, HM, JL, K.

For a Central Pattern cut with $e = 50$ or 40.

PATTERN 19.

$r = 10$ $e = 100$ to 60 $b = 50$ (or 30) from A.

D, CE, BF, [AGN];

cutting BC, DEF, at odd columns only.

$b = 50$ (or 30) from K.

AGN, BFHM, CEJL, DK;

cutting BCDEF at even columns only.

For a Central Pattern cut with $e = 50$ or 40.

PATTERN 20.

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

DK, CEJL, BFHM, [AGN];

cutting DK, CEJL, BFHM, at odd columns only.

$b = 50$ (or 30) from K.

AGN, BFHM, CEJL, DK;

cutting at even columns only.

For a Central Pattern, cut with $e = 50$ or 40.

PATTERN 21.

$r = 10$ $e = 100$ $b = 25$ from A.

AN, BM, CL, DK, EJ, FH, G, FH, EJ, DK, CL, BM, AN.

Plate 28.

PLATE 29.

PLATE 30.

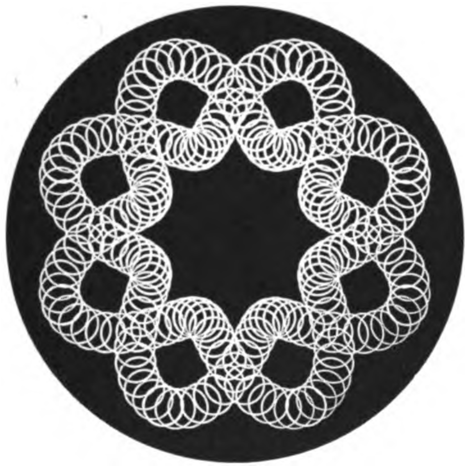
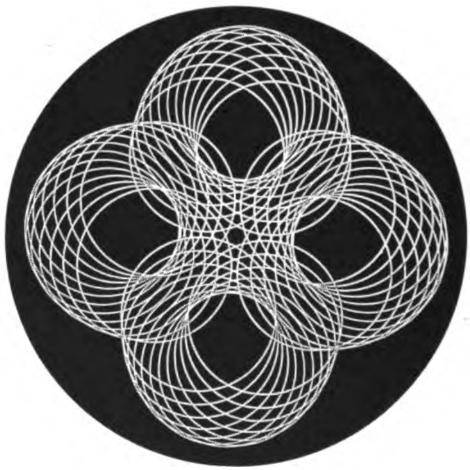


Plate 31.



PATTERN 22.

$r = 40$ $e = 70$ $b = 25$ from A.

A, B, C, D, E, F, G.

PATTERN 23.

$r = 10$ $e = 100$ $b = 50$ from A.

[DN], CE, BF, AG.

$b = 30$ from A, giving $\Delta\ e$ half its tabular value.

DN, M, L, K, J, H.

PATTERN 24.

$r = 10$ $e = 100$ $b = 50$ from K.

[AG], BF, CE, D.

$b = 30$ from K, giving $\Delta\ e$ half its tabular value.

AG, H, J, K, L, M, N.

PATTERN 25.

$r = 10$ $e = 100$ $b = 50$ from A.

[AG], BF, CE, D.

$b = 30$ from A, giving $\Delta\ e$ half its tabular value.

AG, H, J, K, L, M, N.

PATTERN 26.

$r = 15$ $e = 100$ $b = 50$ from G.

G, HF, JE, KD, LC, MB, NA.

PATTERN 27.

$r = 10$ $e = 100$ $b = 60$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N.

PATTERN 28.

$r = 10$ $e = 100$ $b = 50$ from K.

[AG], BF, CE, D.

$b = 30$ from A, giving to Δe half its tabular value.

AG, H, K, L, M, N.

PLATE 32.

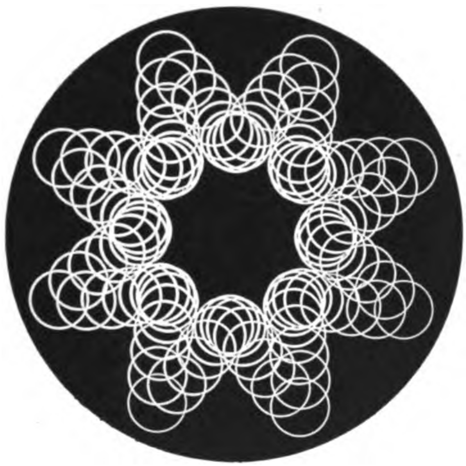


PLATE 33.

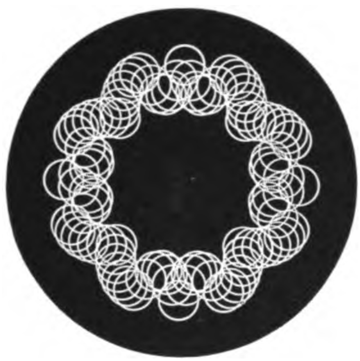
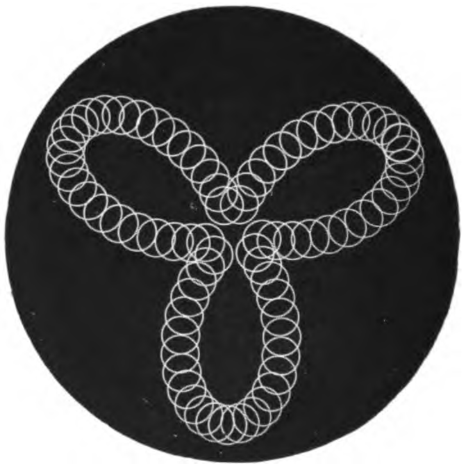


Plate 34.



PART III.

SETTINGS FOR 3-LOOPED FIGURES AND THEIR DERIVATIVES.

PATTERN 1.

$r = 10 \qquad e = 100 \qquad b = 30$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R.

PATTERN 2.

$r = 10 \qquad e = 100 \qquad b = 30$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R.

Some of the last cuts to be cut blind to avoid crowding at centre.

PATTERN 3.

$r = 10 \qquad e = 100 \qquad b = 30$ from A.

AR, BQ, CP, DO, EN, FM, GL, HK, J, HK, GL, FM, EN, DO, CP, BQ, AR.

Or omit the Cuts after J.

PATTERN 4.

$r = 10 \qquad e = 100 \qquad b = 30$ from A.

AR, BQ, CP, DO, EN, FM, GL, HK, J, HK, GL, FM, EN, DO, CP, BQ, AR.

PATTERN 5.

$r = 10$ $e = 100$ $b = 100$ from A.

A, B, C, D, E, F, G, H, J.

Some of the last cuts to be cut blind to avoid confusion.

PATTERN 6.

$r = 10$ $e = 100$ $b = 100$ from A.

AR, BQ, CP, DO, EN, FM, GL, HK, J.

PATTERN 7.

$r = 8$ $e = 70$ $b = 30$ from A.

A, B, C, D, [EN], F, G, H, J.

$b = 30$ from A.

EN, MO, LP, KQ, JR.

PATTERN 8.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, G, H, [JR].

$b = 50$ from N.

JR, KQ, LP, MO, N.

Plate 35.

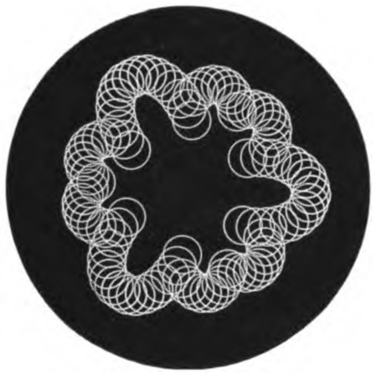


Plate 36.

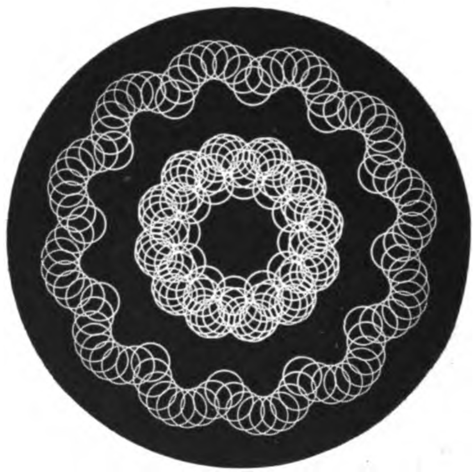
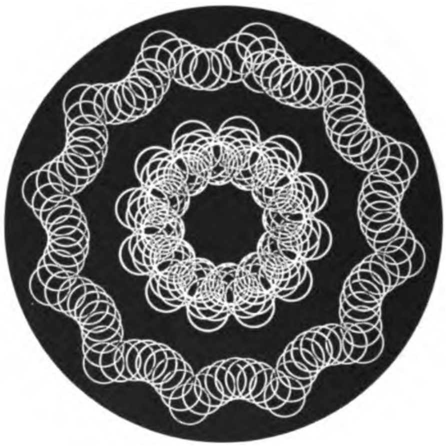


Plate 37.



PATTERN 9.

$r = 10$ $e = 100$ $b = 80$ from A.

N, MO, LP, KQ, [JR].

$b = 80$ from J.

JR, H, G, F, E, D, C, B, A.

PATTERN 10.

$r = 10$ $e = 100$ $b = 80$ from J.

[AJ], K, L, M, N, O, P, Q, R.

$b = 80$ from N.

AJ, BH, CG, DF, E.

PATTERN 11.

$r = 10$ $e = 100$ $b = 80$ (or 30) from A.

EN, DFMO, CGLP, BHKQ, AJR.

For centre, repeat $e = 50$ or 40.

PATTERN 12.

$r = 10$ $e = 100$ $b = 80$ (or 30) from N.

AJR, BHKQ, CGLP, DFMO, EN.

For centre, repeat with $e = 50$ or 40.

PATTERN 13.

$r = 10$ $e = 100$ $b = 80$ from A.

A, B, C, D, E, F, G, H, [JR];

$b = 80$ from A.

JR, KQ, LP, MO, N, cutting at even columns 2, 4, &c. only.

$e = 75$ $b = 80$ from A.

N, MO, LP, KQ, JR;

cutting at odd columns 1, 3, 5, &c. only.

PATTERN 14.

$r = 10$ $e = 100$ $b = 80$ from A.

N, MO, LP, KQ, [JR], cutting at even columns only.

$b = 80$ from J.

[JR], KQ, LP, MO, N.

$b = 80$ from N.

JR, H, G, F, E, D, C, B, A.

PATTERN 15.

$r = 10$ $e = 100$ $b = 80$ (or 30) from A.

EN, DFMO, CGLP, BHKQ, [AJR],

cutting KLMNOPQ at even columns only.

$b = 80$ (or 30) from N.

AJR, KQ, LP, MO, N, cutting at odd columns only.

This may also be cut as a Central Pattern, taking $e = 50$.

PATTERN 16.

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

E, DF, CG, BH, [AJR].

$b = 50$ (or 30) from N.

AJR, KQ, LP, MO, N.

PATTERN 17.

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

E, DF, CG, BH, [AJR], cutting at odd columns only for BCDEFGH.

$b = 50$ (or 30) from N.

AJR, BHKQ, CGLP, DFMO, NE, cutting at odd columns only for BCDEFGH.

PATTERN 18.

$r = 10$ $e = 100$ $b = 50$ (or 30) from A.

EN, DFMO, CGLP, BHKQ, [AJR];

cutting at odd columns only, except for AJR.

$b = 50$ (or 30) from J.

AJR, BHKQ, CGLP, DFMO, EN;

cutting at even columns only.

PATTERN 19.

$r = 40$ $e = 75$ $b = 25$ from A.

A, B, C, D, E, F, G, H, J.

PATTERN 20.

$r = 10$ $e = 50$ $b = 30$ from A.

AR, BQ, CP, DO, EN, FM, GL, HK, J.

PATTERN 21.

$r = 50$ $e = 50$ $b = 30$ from A.

A, B, C, D, E, F, G.

PLATE 38.

PLATE 39.

PLATE 40.

PART IV.

SETTINGS FOR 2-LOOPED FIGURES AND THEIR DERIVATIVES.

(THE GREATER NUMBER OF THESE PATTERNS ARE REMARKABLY BRILLIANT.)

PATTERN 1.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U.

This might also be cut with $e = 80$, all the cuts after N, being omitted.

PATTERN 2.

$r = 10$ $e = 50$ $b = 50$ from A.

AZ, BY, CX, DW, EV, FU, GT, HS, JR, KQ, LP, MO, N.

Where LP, MO, N should be cut blind, to avoid crowding the centre.

PATTERN 3.

$r = 10$ $e = 40$ $b = 30$ from A

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z

This might also be cut with $e = 80$ $b = 50$ (or 60).

PATTERN 4.

$r = 10$ $e = 50$ $b = 25$ from A.

AZ, BY, CX, DW, EV, FU, GT, HS, JR, KQ, LP, MO, N.

PATTERN 5.

$r = 10$ $e = 50$ $b = 25$ from N.

N, OM, PL, QK, RJ, SH, TG, UF, VE, WD, XC, YB, ZA.

PATTERN 6.

$r = 10$ $e = 50$ $b = 25$ from A.

A, B, C, D, E, F, [GT], H, J, K, L, M, N.

$b = 25$ from A.

GT, SU, RV, QW, PX, OY, Z.

The cuts, L, M, N, may be cut blind.

PATTERN 7.

$r = 10$ $e = 80$ or 70 $b = 40$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, [NZ].

$b = 40$ from T.

NZ, OY, PX, QW, RV, SU, T.

Plate 41.

Plate 42.

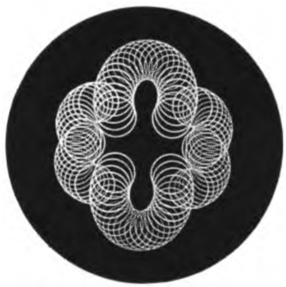


PLATE 43.

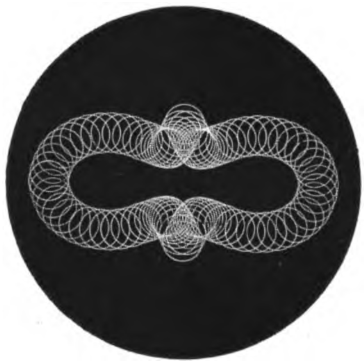


PLATE 44.

Plate 45.

Plate 46.

Plate 47.

PATTERN 8.

$r = 10$ $e = 80$ or 70 $b = 40$ from A.

G, HF, JE, KD, LC, MB, [NA].

$b = 40$ from N.

NA, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 9.

$r = 10$ $e = 80$ to 70 $b = 40$ from N.

[NA], O, P, Q, R, S, T, U, V, W, X, Y, Z.

$b = 40$ from T.

NA, MB, LC, KD, JE, HF, G.

PATTERN 10.

$r = 10$ $e = 80$ to 40 $b = 40$ from A.

GT, FHSU, EJRV, DKQW, CLPX, BMOY, ANZ.

PATTERN 11.

$r = 10$ $e = 80$ to 40 $b = 40$ from T.

ANZ, BMOY, CLPX, DKQW, EJRV, FHSU, GT.

PATTERN 12.

$r = 10$ $e = 80$ $b = 40$ from A.

T, SU, RV, QW, PX, OY, [NZ], cutting in even columns only, (except NZ).

$b = 40$ from T, cutting in odd columns only.

[NZ], OY, PX, QW, RV, SU, T.

$b = 40$ from N.

NZ, M, L, K, J, H, G, F, E, D, C, B, A.

PATTERN 13.

$r = 13$ $e = 80$ to 40 $b = 40$ from A.

GT, FHSU, EJRV, DKQW, CLPX, BMOY, [ANZ]; cutting T, SU, RV, QW, PX, OY, in odd columns only.

$b = 40$ from T.

ANZ, OY, PX, QW, RV, SU, T; cutting in even columns only.

PATTERN 14.

$r = 10$ $e = 80$ to 40 $b = 40$ from A.

G, FH, EJ, DK, CL, BM, [ANZ].

$b = 40$ from T.

ANZ, OY, PX, QW, RV, SU, T.

PATTERN 15.

$r = 10$ $e = 80$ to 40 $b = 40$ from A.

G, FH, EJ, DK, CL, BM, [ANZ];

cutting G, FH, EJ, DK, CL, BM, in odd columns only.

$b = 40$ from T.

ANZ, OYBM, PXCL, QWDK, RVEJ, SUFH, TG;

cutting G, FH, EJ, DK, CL, BM, in even columns only.

PATTERN 16.

$r = 10$ $e = 80$ to 40 $b = 40$ from A.

GT, FHSU, EJRV, DKQW, CLPX, BMOY, [ANZ];

cutting in odd columns only, (except ANZ).

$b = 40$ from T.

ANZ, BMOY, CLPX, DKQW, EJRV, FHSU, GT;

cutting in even columns only.

PATTERN 17.

$r = 40$ $e = 70$ $b = 80$ from A.

A, B, C, D, E.

PATTERN 18.

$r = 10$ $e = 80$ $b = 40$ from A.

G, HF, JE, KD, LC, MB, NA, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 19.

$r = 10 \qquad e = 80 \qquad b = 40$ from A.

G, HF, JE, KD, LC, MB, [NA].

$b = 40$ from N.

NA, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 20.

$r = 10 \qquad e = 80 \qquad b = 40$ from A.

[GZ], FH, EJ, DK, CL, BM, AN.

$b = 25$ from A, giving Δe half its tabular value.

GZ, Y, W, V, U, T, S, R, Q, P, O.

PATTERN 21.

$r = 10 \qquad e = 80 \qquad b = 50$ from T.

[AN], BM, CL, DK, EJ, FH, GZ.

$b = 25$ from N, giving Δe half its tabular value.

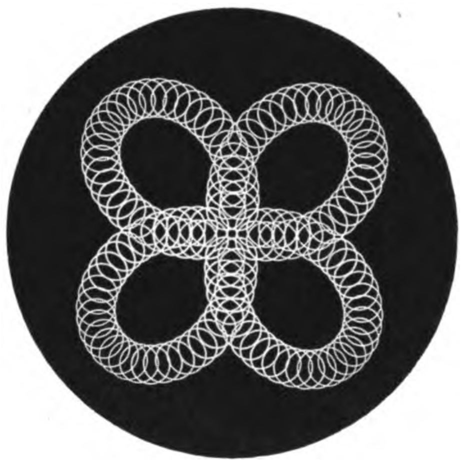
AN, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 22.

$r = 10 \qquad e = 100 \qquad b = 50$ from A.

AZ, BY, CX, DW, EV, FU, GT, HS, JR, KQ, LP, MO, N, MO, LP, [KQ], KQ, JR, HS, GT, FU, EV.

PLATE 48.



PATTERN 23.

$r = 10$ $e = 80$ $b = 40$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, [NZ].

$b = 40$ from T, cutting in even columns only.

NZ, OY, PX, QW, RV, SU, T.

$e = 55$ $b = 40$ from A, cutting in odd columns only.

T, SU, RV, QW, PX, OY.

PART V.

SETTINGS FOR $\frac{3}{2}$-LOOPED FIGURES.

THE reader may possibly have some little difficulty in conceiving to himself what a fractional loop can be.

Bearing in mind that once cutting round the division-plate in a 5-looped or 4-looped figure, gives 5 or 4 loops or waves respectively, he will have no difficulty in perceiving that cutting once round the division-plate in a $\frac{3}{2}$-looped figure will give $1\frac{1}{2}$ loops. In order to complete the figure, we go on cutting round till we complete an integral number of loops at the end of an integral number of revolutions.

PATTERN 1.

$r = 10$ $e = 100$ $b = 30$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, [Y], Y, Z, A^2, B^2.

PATTERN 2.

$r = 10$ $e = 100$ $b = 25$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, [Y], Y, Z, A^2, B^2, C^2, D^2, E^2, F^2, G^2, H^2.

Plate 49.

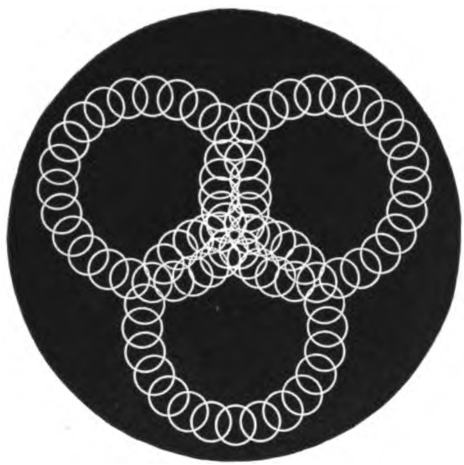


Plate 50.

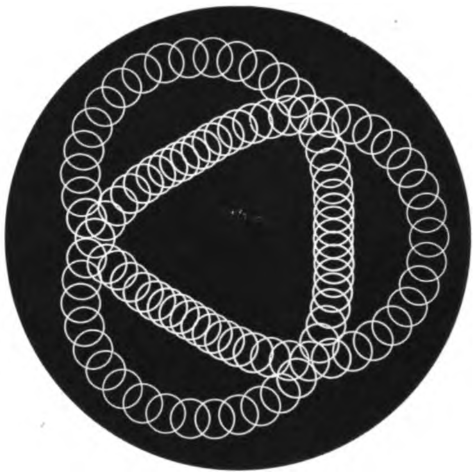


Plate 51.

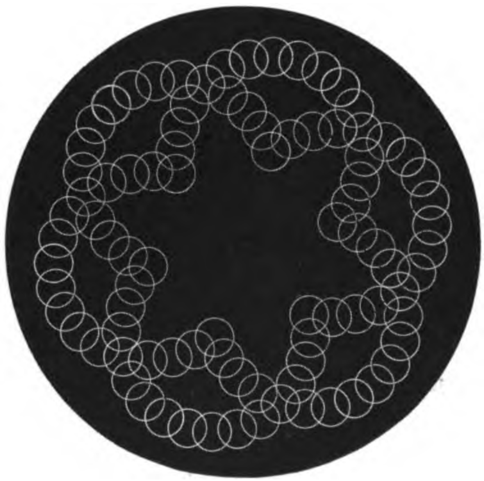
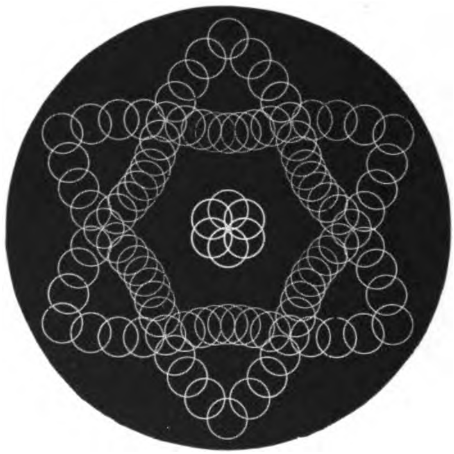


Plate 52.



PATTERN 3.

$r = 10$ $e = 100$ to 60 $b = 80$ from A.

AR, BQ, CP, DO, EN, FM, GL, HK, J, KH, LG, MF, NE, OD, PC, QB, RA.

PATTERN 4.

$r = 10$ $e = 100$ $b = 80$ from J.

AR, BQ, CP, DO, EN, FM, GL, HK, J, KH, LG, MF, NE, OD, PC, QB, RA.

PATTERN 5.

$r = 20$ $e = 80$ $b = 80$ from A.

R, S, T, U, V, W, X, Y, Z, A^2, B^2, C^2, D^2, E^2, F^2, G^2, H^2.

PART VI.

SETTINGS FOR $\frac{5}{2}$-LOOPED FIGURES.

PATTERN 1.

$r = 10$ $e = 100$ $b = 50$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z.

Cut some of last cuts blind.

PATTERN 2.

$r = 10$ $e = 100$ $b = 25$ from A.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 3.

$r = 10$ $e = 100$ $b = 50$ from A.

AN, BM, CL, DK, EJ, FH, G, HF, JE, KD, LC, MB, NA.

PLATE 53.

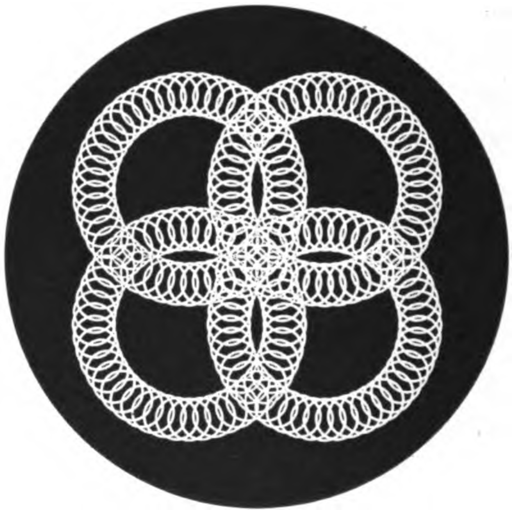


Plate 54

PART VII.

SETTINGS FOR CIRCULAR FIGURES.

PATTERN 1.

$d = 100$ or 60 $r = 10$.

A to Z. TABLE I.

PATTERN 2.

$d = 80$ $r = 10$.

A to Z. TABLE III.

PATTERN 3.

$d = 50$ $r = 10$.

A to N. TABLE I.

$d = 75$

N to Z. TABLE II.

Either the centre or border of this Pattern may be used separately, compare Patterns 4 and 5.

PATTERN 4.

$d = 80 \qquad r = 10.$

A to J. TABLE III.

Compare the first part of Pattern 3.

PATTERN 5.

$d = 75 \qquad r = 10.$

N to R. TABLE III.

Compare the second part of Pattern 3.

PATTERN 6.

$d = 50 \qquad r = 10.$

A to N. TABLE I., cutting in column 1 only.

PATTERN 7.

$d = 75 \qquad r = 10.$

A to N. TABLE III., cutting in column 1 only

PATTERN 8.

$d = 100$ or $80 \qquad r = 10.$

A to G. TABLE I., cutting in column 1 only.

A to G. TABLE II., cutting in column 2 only.

When this Pattern is cut with a small value of d, it forms a pretty little centre piece.

Plate 55.

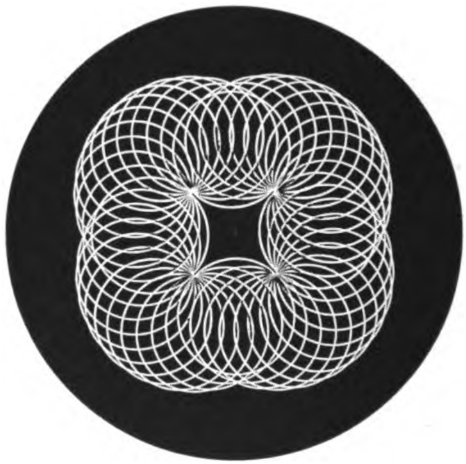
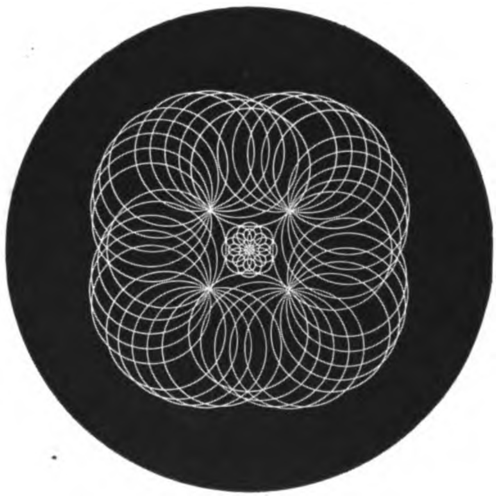


Plate 56.



PATTERN 9.

If $r = \frac{d}{2}$ each cut will pass through the centre of the circle on which the cuts lie, forming a Turk's-head or part of a Turk's-head; which if completed will just touch the centre of the work. In most cases it is desirable not to complete any one of the Turk's-heads, but to commence with Z, the cut furthest from the centre of the work, and to continue the cuts until the same cut has been made in two adjacent Turk's-heads.

Turk's-heads cut in this manner form a very fine Pattern, and afford a test of the accuracy of the workman, as the slightest error will show itself by the circles composing each Turk's-head not meeting accurately in the centre of it.

$d = 60 \qquad r = 30.$

Z to N. TABLE I., or omit the cuts given by each alternate letter.

PATTERN 10.

$d = 60 \qquad r = 30.$

Z to R. TABLE III., or omit the alternate cuts.

PATTERN 11.

$d = 60 \qquad r = 30.$

Z, X, V, T, R, P, N. TABLE I.

Cutting X and P in column 1 and T in column 2 only. Compare Engleheart, No. 2.

PATTERN 12.

$d = 60$ $r = 30.$

Z, W, T, R, N. TABLE I.

PATTERN 13.

$d = 60$ $r = 30.$

Z, X, T, R. TABLE III.

PATTERN 14.

The cuts in the circles can be arranged in shells. It is curious to observe the manner in which each shell appears to point towards the centre of the work.

$d = 80$ $r = 20$ TABLE I.

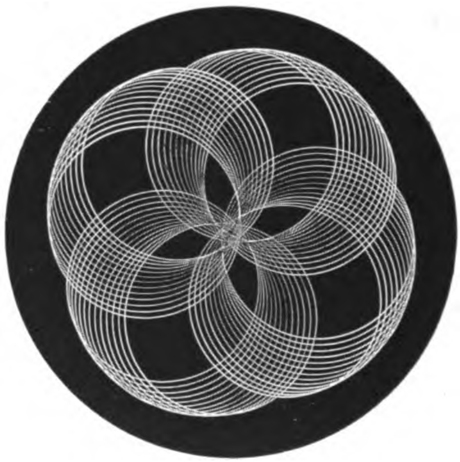
Z, R, J, A.

Place the circles Z and R accurately in contact (*ante*, p. 102).

Diminish the radius by 2, and cut the Pattern over again with eccentricities less by 2 than the tabular eccentricities. Diminish the radius by 2 more, 4 in all, and cut with eccentricities less by 4 than the tabular eccentricities. Repeat the process of diminishing the radius and cutting with eccentricities less than the tabular eccentricities, the total quantity by which the radius is diminished from 20 in any one cut, being equal to the quantity by which the tabular eccentricity is diminished for the same cut.

It is best to make the cuts A in one pair of columns only, to avoid confusion.

Plate 57.



PATTERN 15 (Shell Pattern).

$d = 80$ $r = 15\frac{1}{2}$ TABLE I.

D, K, Q, W.

Repeat with the following values of r; viz., $14\frac{1}{2}$, $13\frac{1}{2}$, $9\frac{1}{2}$, $8\frac{1}{2}$, $7\frac{1}{2}$, $3\frac{1}{2}$, $2\frac{1}{2}$, $1\frac{1}{2}$; the value of e for each cut being as much less than the tabular value as the value of r is less than $15\frac{1}{2}$.

PATTERN 16.

$d = 100$ $r = 19{\cdot}5$ TABLE I.

A, D, G, K, N, R, T, W, Z.

Repeat diminishing the value of r and the tabular value of e by 2 each time.

PATTERN 17.

$d = 100$ $r = 9{\cdot}8$.

Z, W, T, Q, N.

Repeat diminishing the value of r and the tabular value of e by $2\frac{1}{2}$ each time.

PATTERN 18.

$d = 80$ $r = 50$.

T to Z.

PATTERN 19.

$d = 50 \qquad r = 25.$

T to N.

PATTERN 20.

$d = 100 \qquad r = 100$

T to Z.

PLATE 58.

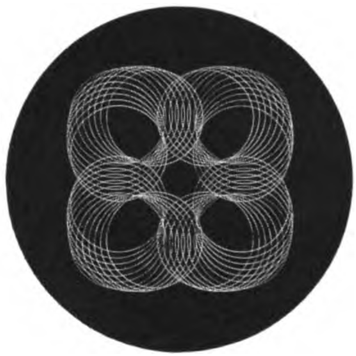


Plate 59.

PATTERN 21.

THE CARDIOID.

Cut with $r = 10$ at the numbers with the eccentricities given below, taking the eccentricities from either column I. or column II.

Number on Scale 96.		Eccentricities.		Number on Scale 96.		Eccentricities.	
		I.	II.			I.	II.
96		200	100				
1	95	199·8	100	25	71	93·5	46·8
2	94	199·1	99·5	26	70	86·9	43·5
3	93	198·1	99	27	69	80·5	40·3
4	92	196·6	98·3	28	68	74·1	37
5	91	194·7	97·3	29	67	67·9	34·1
6	90	192·4	96·2	30	66	61·7	30·8
7	89	189·7	94·6	31	65	55·7	27·9
8	88	186·6	93·3	32	64	50	25
9	87	183·1	91·5	33	63	44·4	22·2
10	86	179·3	89·6	34	62	39·1	19·6
11	85	175·2	87·7	35	61	34·1	17
12	84	170·7	85·3	36	60	29·3	14·6
13	83	165·9	82·9	37	59	24·8	12·4
14	82	160·9	80·5	38	58	20·7	10·4
15	81	155·6	77·8	39	57	16·9	8·5
16	80	150	75	40	56	13·4	6·7
17	79	144·2	72·1	41	55	10·3	5·1
18	78	138·3	69·1	42	54	7·6	3·8
19	77	132·1	66	43	53	5·3	2·6
20	76	125·9	62·9	44	52	3·4	1·7
21	75	119·5	59·7	45	51	2	1
22	74	113·1	56·5	46	50	1	½
23	73	106·5	53·2	47	49	¼	0
24	72	100	50	48		0	0

Two cardioids back to back make a fine pattern

PART VIII.

SETTINGS FOR STRAIGHT LINE FIGURES.

PATTERN 1.

AN EQUILATERAL TRIANGLE.

$r = 10$ $p = 80$ from TABLE I.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R.

To be cut from the Table for 3-looped figures.

PATTERN 2.

THE HEXAGON.

$r = 10$ $p = 80$ from TABLE I.

AR, BQ, CP, DO, EN, FM, GL, HK, J.

To be cut from the Table for 3-looped figures.

PATTERN 3.

THE SQUARE.

$r = 10$ $p = 60$ from TABLE I.

A, B, C, D, E, F, G, H, J, K, L, M, N.

To be cut from the Table for 4-looped figures.

Plate 60.

PATTERN 4.

THE OCTAGON.

$r = 10$ $p = 80$ from Table I

AN, BM, CL, DK, EJ, FH, G.

To be cut from the Table for 4-looped figures.

PATTERN 5.

THE PENTAGON.

$r = 10$ $p = 60$ from Table II.

A, B, C, D, E, F, G, H, J, K, L, M, N.

To be cut from the Table for 5-looped figures.

PATTERN 6.

THE DECAGON.

$r = 10$ $p = 50$ from Table II.

AN, BM, CL, DK, EJ, FH, G.

To be cut from the Table for 5-looped figures.

PATTERN 7.

TWO TRIANGLES, BACK TO BACK.

$r = 10$ $p = 50$ from Table I.

AR, BQ, CP, DO, EN, FM, GL, HK, J, KH, LG, MF, NE, OD, PC, QB, RA.

To be cut from the Table for 3-looped figures.

PATTERN 8.

$r = 10$ $p = 60$ from Table I.

AN, BM, CL, DK, EJ, FH, G, HF, JE, KD, LC, MB, NA.

To be cut from the Table for 4-looped figures.

PATTERN 9.

$r = 10$ $p = 60$ from Table II.

AN, BM, CL, DK, EJ, FH, G, HF, JE, KD, LC, MB, NA.

To be cut from the Table for 5-looped figures.

PATTERN 10.

A SOLID TRIANGLE.

Cut Pattern 1 with the following values of p; viz. 60, 40, 20.

The centre may be filled up with a small shell or Turk's-head. The workman must be on his guard to avoid loss of time.

PATTERN 11.

A SOLID HEXAGON.

Cut Pattern 2 with the following values of p; viz. 80, 60, 40, 20.

The centre may be filled up with a small shell or Turk's-head. The workman must be on his guard against loss of time.

PATTERN 12.

A SOLID SQUARE.

Cut Pattern 3 with the following values of p; viz. 60, 40, 20.

The centre may be filled up with a small shell or Turk's-head. The workman must be on his guard against loss of time.

PATTERN 13.

A SOLID OCTAGON.

Cut Pattern 4 with the following values of p; viz. 80, 60, 40, 20.

The centre may be filled up with a small shell or Turk's-head. The workman must be on his guard against loss of time.

PATTERN 14.

A SOLID PENTAGON.

Cut Pattern 5 with the following values of p; viz. 60, 40, 20.

The centre may be filled up with a small shell or Turk's-head. The workman must be on his guard against loss of time.

PATTERN 15.

$r = 23$ $\quad p = 30$ from TABLE I.

G, J, L, N.

To be cut from the Table for 4-looped figures, cutting at the even columns only, placing the cuts G accurately in contact.

PATTERN 16.

$r = 29\frac{3}{4}$ $p = 30$ from TABLE I.

J, L, N, P, R.

To be cut from the Table for 4-looped figures, cutting at the even columns only, placing the cuts J accurately in contact.

PATTERN 17.

THE LETTER H.

$r = 10$ $p = 40$ from TABLE I.

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R.

To be cut from the Table for 2-looped figures.

Replace the division-plate and slide-rest into the position for the cut at number 4, and, without moving the division-plate, make cuts, moving the slide-rest backwards $\frac{1}{2}$ between each cut, till the horizontal stroke of the letter is completed.

The exercise of a little ingenuity will enable the workman to cut several of the letters which are formed of straight lines only.

PLATE 61.

PLATE 63.

PART IX.

SETTINGS FOR THE ELLIPSE.

PATTERN 1.

$r = 10 \qquad a = 100.$

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 2.

$r = 10 \qquad a = 100.$

AZ, BY, CX, DW, EV, FU, GT, HS, JR, KQ, LP, MO, N.

PATTERN 3.

$r = 10 \qquad a = 100.$

N, OM, PL, QK, RJ, SH, TG, UF, VE, WD, XC, YB, ZA.

PATTERN 4.

Cut both Patterns 2 and 3.

PATTERN 5.

$r = 30 \qquad a = 50.$

A, B, C, D, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z.

PATTERN 6.

$r = 17 \qquad a = 90.$

A, D, G, K, N, Q, T, W, Z.

Repeat the cuts with $r = 15$ and $e = 2$ less than tabular value.

Repeat the process till the shells are completed. The circles at 10, 14, should be placed accurately in contact by a proper adjustment of r.

PATTERN 7.

"THE DOUBLE SEA HEDGEHOG."

$r = 40 \qquad a = 80.$

AZ, BY, CX, &c.

The cuts must be continued till they nearly meet in the centre of the work.

PATTERN 8.

A WAVED ELLIPSE.

This may be cut in the manner following. Calculate the eccentricities for each cut of Pattern 9 or Pattern 12 in the settings for 4-looped figures and their derivatives (*ante*, p. 143), subtract the value of the eccentricity for each cut from 100; subtract the difference from the eccentricity given in the Table for the corresponding cuts in the Ellipse; or it may be cut from the Tables for waved Ellipses.

Plate 64.

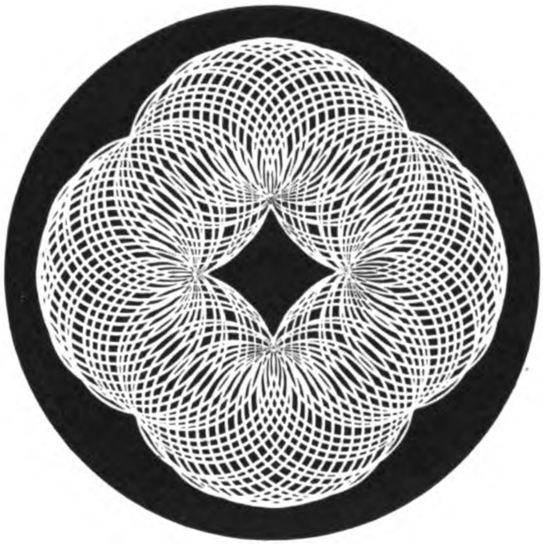


Plate 66.

SETTINGS FOR ENVELOPES.

I.—SETTINGS FOR PATTERNS CUT ACCORDING TO THE METHOD DESCRIBED IN CHAPTER V., ART. 3.

PATTERN 1.

THE EQUILATERAL TRIANGLE.

Cut at 96, 32, 64 on the Scale of 96, with the following corresponding values of e and r:—

e = 100	90	80	70	60	50	40	30	20	10	0
r = 0	5	10	15	20	25	30	35	40	45	50

PATTERN 2.

THE SQUARE.

Cut at 96, 24, 48, 72 on the Scale of 96 with the following corresponding values of e and r:—

e = 100	90	80	70	60	50	40	30	20	10	0
r = 0	7·07	14·14	21·21	28·28	35·35	42·42	49·49	56·56	63·63	70·7
or r = 0	7	14¼	21¼	28¼	35¼	42½	49½	56½	63½	70¾

PATTERN 3.

THE PENTAGON.

Cut at 120, 24, 48, 72, 96 on the Scale of 120, with the following corresponding values of *e* and *r* :—

e = 100	90	80	70	60	50	40	30	20	10	0
r = 0	8·09	16·18	24·27	32·36	40·45	48·54	56·63	64·72	72·81	80·9
or r = 0	8	$16\frac{1}{4}$	$24\frac{1}{4}$	$32\frac{1}{4}$	$40\frac{1}{2}$	$48\frac{1}{2}$	$56\frac{3}{4}$	$64\frac{3}{4}$	$72\frac{3}{4}$	81

PATTERN 4.

THE HEXAGON.

Cut at 96, 16, 32, 48, 64, 80 on the Scale of 96, with the following corresponding values of *e* and *r* :—

e = 100	90	80	70	60	50	40	30	20	10	0
r = 0	8·66	17·32	25·98	34·64	43·30	51·96	60·62	69·28	77·94	86·60
or r = 0	$8\frac{3}{4}$	$17\frac{1}{4}$	3	$34\frac{3}{4}$	$43\frac{1}{4}$	52	$60\frac{1}{2}$	$69\frac{1}{4}$	78	$80\frac{1}{2}$

PATTERN 5.

A SIX-POINTED STAR.

Cut with the radius and eccentricity given for the Equilateral Triangle (*ante*, Pattern 1,) at 96, 16, 32, 48, 64, 80.

Plate 67.

II.—SETTINGS FOR PATTERNS CUT ACCORDING TO THE METHOD DESCRIBED IN CHAPTER V., ART. 5.

PATTERN 6.

THE SQUARE.

Cut from the TABLE FOR CIRCULAR FIGURES, No. I., with the proper eccentricity for each cut when $d = 100$ with the radii given below:—

	A	B	C	D	E	F	G	H	J	K	L	M	N
$r =$	50	43·45	37·05	30·85	25·00	19·55	14·65	10·35	6·70	3·80	1·70	·45	0
or $r =$	50	43½	37	31	25	19½	14¾	10½	6¾	3¾	1¾	½	0

PATTERN 7.

THE HEXAGON.

Cut from the TABLE FOR CIRCULAR FIGURES, No. III., with the proper eccentricity for each cut when $d = 100$ with the radii given below:—

	A	B	C	D	E	F	G	H	J
$r =$	50	43·45	37·05	30·85	25	19·55	14·65	10·35	6·70
or $r =$	50	43½	37	31	25	19½	14¾	10½	6¾

III.—SETTINGS FOR PATTERNS CUT ACCORDING TO THE METHOD DESCRIBED IN CHAPTER V., ART. 7.

PATTERN 8.

Cut from the TABLE FOR 2-LOOPED FIGURES with a constant eccentricity $e = 50$, with following values of r :—

A	B	C	D	E	F	G	H	J	K	L
$r = 0$	3¼	6½	9¾	13	16	19	22	25	27¾	30½

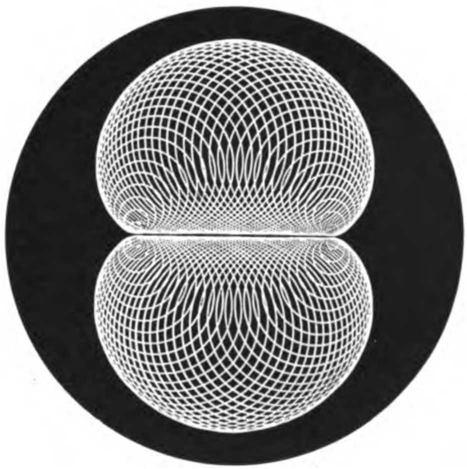
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
$r = 3$	35¼	37¾	39¾	41½	43¼	44¾	46	47¼	48¼	49	49½	49¾	50

PATTERN 9.

Cut from the TABLE FOR CIRCULAR FIGURES, No. I., $d = 100$, with the proper eccentricities and the following values of r :—

Z	Y	X	W	V	U	T	S	R	Q	P	O	N
$r = 50$	49½	48¼	46	43¼	39¾	35¼	30¼	25	19	13	6½	0

PLATE 68.



MISCELLANEOUS PATTERNS.

PATTERN 10.

$e = 50$, cut from Table for 4-Looped Figures with the following values of r:—

A	B	C	D	E	F	G	H	J	K	L	M	N
$r = 50$	$43\frac{1}{3}$	37	$30\frac{3}{4}$	25	$19\frac{1}{2}$	$14\frac{1}{2}$	$10\frac{1}{4}$	$6\frac{3}{4}$	$3\frac{3}{4}$	$1\frac{1}{3}$	$\frac{1}{3}$	0

PATTERN 11.

Cut from Table for Circular Figures, No. I. and No. II., $d = 100$ with the proper eccentricities and the following values of r:—

Z	Y	X	W	V	U	T
$r = 50$	37	25	$14\frac{1}{2}$	$6\frac{3}{4}$	$1\frac{1}{4}$	0

LIST OF THE TABLES.

TABLE FOR CONTACT OF CIRCLES.—N°. I.

Number of Circles.	Modulus.	Number of Circles.	Modulus.
2	1·000	32	·097
3	·866	36	·087
4	·707	40	·078
5	·588	45	·070
6	·5	48	·065
7	·434	56	·056
8	·383	60	·052
9	·342	64	·049
10	·309	72	·044
12	·259	90	·035
14	·223	96	·033
15	·208	112	·028
16	·195	120	·026
18	·174	144	·022
20	·156	180	·017
24	·131	192	·016
28	·112	360	·009
30	·105		

TABLE FOR CONTACT OF CIRCLES.—N°. II.

n =	3	4	6	8	12	16	18	24
e = 100	86·6	70·7	50	38·3	26	19·5	17·4	13·1
90	77·9	63·6	45	34·4	23·4	17·6	15·7	11·8
80	69·2	56·6	40	30·7	20·8	15·6	13·9	10·5
70	60·6	49·5	35	26·8	18·2	13·6	12·2	9·2
60	51·9	42·4	30	23·0	15·6	11·7	10·4	7·9
50	43·3	35·3	25	19·1	13	9·8	8·7	6·6
40	34·6	28·3	20	15·4	10·4	7·8	6·9	5·2
30	25·9	21·2	15	11·5	7·8	5·9	5·2	3·9
20	17·3	14·1	10	7·7	5·2	3·9	3·5	2·6
10	8·6	17·0	5	3·8	2·6	20	1·7	1·3
Δ	·87	·71	·5	·38	·26	·2	·17	·13

TABLE FOR CONTACT OF CIRCLES.—No. III.

$n =$	10	12	14	16	24	32
$r =$ 8	25·6	30·88	35·84	41·06	61·07	82·48
12	38·4	46·32	53·76	61·59	91·60	123·72
16	51·2	61·76	71·68	82·12	122·14	164·96
20	64	77·20	89·60	102·65	152·67	206·20
24	76·8	92·64	107·52	123·18	183·20	
28	89·6	108·08	125·44	143·72	213·73	
32	102·4	123·52	143·36	164·24		
36	115·2	138·96	161·28	184·77		
40	128	154·40	179·20	205·33		
44	140·8	169·84	187·12			
48	153·6	185·28	215·04			

TABLES FOR 5-LOOPED FIGURES AND THEIR DERIVATIVES.—No. I.

$(e = a + b \sin 5\theta.)$

DIVISION-PLATE, SCALE 120.

	1	2	3	4	5	6	7	8	9	10
A	6	...	30	...	54	...	78	...	102	...
B	5	7	29	31	53	55	77	79	101	103
C	4	8	28	32	52	56	76	80	100	104
D	3	9	27	33	51	57	75	81	99	105
E	2	10	26	34	50	58	74	82	98	106
F	1	11	25	35	49	59	73	83	97	107
G	120	12	24	36	48	60	72	84	96	108
H	119	13	23	37	47	61	71	85	95	109
J	118	14	22	38	46	62	70	86	94	110
K	117	15	21	39	45	63	69	87	93	111
L	116	16	20	40	44	64	68	88	92	112
M	115	17	19	41	43	65	67	89	91	113
N	...	18	...	42	...	66	...	90	...	114

TABLES FOR 5-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. II.

DIFFERENCES OF ECCENTRICITY.

VALUES OF Δe, when							
$b =$	100	75	60	50	40	30	25
A	3	$2\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$
B	$10\frac{1}{2}$	$7\frac{3}{4}$	6	$5\frac{1}{4}$	4	3	$2\frac{1}{2}$
C	16	12	$9\frac{1}{2}$	8	$6\frac{1}{4}$	$4\frac{3}{4}$	4
D	$20\frac{1}{2}$	$15\frac{1}{4}$	$12\frac{1}{2}$	$10\frac{1}{4}$	$8\frac{1}{4}$	$6\frac{1}{4}$	5
E	24	18	$14\frac{1}{2}$	12	$9\frac{3}{4}$	$7\frac{1}{4}$	6
F	26	$19\frac{3}{4}$	$15\frac{1}{2}$	13	$10\frac{1}{2}$	$7\frac{3}{4}$	$6\frac{3}{4}$
G	26	$19\frac{3}{4}$	$15\frac{1}{2}$	13	$10\frac{1}{2}$	$7\frac{3}{4}$	$6\frac{3}{4}$
H	24	18	$14\frac{1}{2}$	12	$9\frac{3}{4}$	$7\frac{1}{4}$	6
J	$20\frac{1}{2}$	$15\frac{1}{4}$	$12\frac{1}{2}$	$10\frac{1}{4}$	$8\frac{1}{4}$	$6\frac{1}{4}$	5
K	16	12	$9\frac{1}{2}$	8	$6\frac{1}{4}$	$4\frac{3}{4}$	4
L	$10\frac{1}{2}$	$7\frac{3}{4}$	6	$5\frac{1}{4}$	4	3	$2\frac{1}{2}$
M	3	$2\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$

TABLES FOR 4-LOOPED FIGURES AND THEIR DERIVATIVES.—No. I.

($e = a + b \sin 4\theta$.)

Division-Plate, Scale 96.

	1	2	3	4	5	6	7	8
A	6	...	30	...	54	...	78	...
B	5	7	29	31	53	55	77	79
C	4	8	28	32	52	56	76	80
D	3	9	27	33	51	57	75	81
E	2	10	26	34	50	58	74	82
F	1	11	25	35	49	59	73	83
G	96	12	24	36	48	60	72	84
H	95	13	23	37	47	61	71	85
J	94	14	22	38	46	62	70	86
K	93	15	21	39	45	63	69	87
L	92	16	20	40	44	64	68	88
M	91	17	19	41	43	65	67	89
N	...	18	...	42	...	66	...	90

TABLES FOR 4-LOOPED FIGURES AND THEIR DERIVATIVES.—No. II.

DIFFERENCES OF ECCENTRICITY.

	Values of Δe, when							
b =	100	80	75	60	50	40	30	25
A	3	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$
B	$10\frac{1}{2}$	$8\frac{1}{2}$	$7\frac{3}{4}$	6	$5\frac{1}{4}$	4	3	$2\frac{1}{2}$
C	16	$12\frac{3}{4}$	12	$9\frac{1}{2}$	8	$6\frac{1}{4}$	$4\frac{3}{4}$	4
D	$20\frac{1}{2}$	$16\frac{1}{2}$	$15\frac{1}{4}$	$12\frac{1}{2}$	$10\frac{1}{4}$	$8\frac{1}{4}$	$6\frac{1}{4}$	5
E	24	$19\frac{1}{4}$	18	$14\frac{1}{2}$	12	$9\frac{3}{4}$	$7\frac{1}{4}$	6
F	26	$20\frac{1}{2}$	$19\frac{3}{4}$	$15\frac{1}{2}$	13	$10\frac{1}{2}$	$7\frac{3}{4}$	$6\frac{3}{4}$
G	26	$20\frac{1}{2}$	$19\frac{3}{4}$	$15\frac{1}{2}$	13	$10\frac{1}{2}$	$7\frac{3}{4}$	$6\frac{3}{4}$
H	24	$19\frac{1}{4}$	18	$14\frac{1}{2}$	12	$9\frac{3}{4}$	$7\frac{1}{4}$	6
J	$20\frac{1}{2}$	$16\frac{1}{2}$	$15\frac{1}{4}$	$12\frac{1}{2}$	$10\frac{1}{4}$	$8\frac{1}{4}$	$6\frac{1}{4}$	5
K	16	$12\frac{3}{4}$	12	$9\frac{1}{2}$	8	$6\frac{1}{4}$	$4\frac{3}{4}$	4
L	$10\frac{1}{2}$	$8\frac{1}{2}$	$7\frac{3}{4}$	6	$5\frac{1}{4}$	4	3	$2\frac{1}{2}$
M	3	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$

TABLES FOR 3-LOOPED FIGURES AND THEIR DERIVATIVES.—No. I.

$(\varrho = a + b \sin 3\theta.)$

DIVISION-PLATE, SCALE 96.

	1	2	3	4	5	6
A	8	...	40	...	72	...
B	7	9	39	41	71	73
C	6	10	38	42	70	74
D	5	11	37	43	69	75
E	4	12	36	44	68	76
F	3	13	35	45	67	77
G	2	14	34	46	66	78
H	1	15	33	47	65	79
J	96	16	32	48	64	80
K	95	17	31	49	63	81
L	94	18	30	50	62	82
M	93	19	29	51	61	83
N	92	20	28	52	60	84
O	91	21	27	53	59	85
P	90	22	26	54	58	86
Q	89	23	25	55	57	87
R	...	24	...	56	...	88

TABLES FOR 3-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. II.

Differences of Eccentricity.

Values of Δe, when								
$b =$	100	80	75	60	50	40	30	25
A	2	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
B	$5\frac{3}{4}$	$4\frac{1}{2}$	4	$3\frac{1}{2}$	$2\frac{3}{4}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{4}$
C	$9\frac{1}{4}$	$7\frac{1}{2}$	$6\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{1}{4}$
D	$12\frac{1}{2}$	10	$9\frac{3}{4}$	$7\frac{1}{2}$	$6\frac{1}{2}$	5	$3\frac{3}{4}$	$3\frac{1}{4}$
E	15	12	$11\frac{1}{4}$	9	$7\frac{1}{2}$	6	$4\frac{1}{2}$	$3\frac{3}{4}$
F	$17\frac{1}{4}$	14	$13\frac{1}{4}$	$10\frac{1}{4}$	$8\frac{3}{4}$	7	$5\frac{1}{4}$	$4\frac{1}{2}$
G	$18\frac{3}{4}$	15	14	$11\frac{1}{4}$	$9\frac{1}{4}$	$7\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{3}{4}$
H	$19\frac{1}{2}$	$15\frac{1}{2}$	$14\frac{1}{2}$	$11\frac{3}{4}$	$9\frac{3}{4}$	$7\frac{3}{4}$	6	$4\frac{3}{4}$
J	$19\frac{1}{2}$	$15\frac{1}{2}$	$14\frac{1}{2}$	$11\frac{3}{4}$	$9\frac{3}{4}$	$7\frac{3}{4}$	6	$4\frac{3}{4}$
K	$18\frac{3}{4}$	15	14	$11\frac{1}{4}$	$9\frac{1}{4}$	$7\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{3}{4}$
L	$17\frac{1}{4}$	14	$13\frac{1}{4}$	$10\frac{1}{4}$	$8\frac{3}{4}$	7	$5\frac{1}{4}$	$4\frac{1}{2}$
M	15	12	$11\frac{1}{4}$	9	$7\frac{1}{2}$	6	$4\frac{1}{2}$	$3\frac{3}{4}$
N	$12\frac{1}{2}$	10	$9\frac{3}{4}$	$7\frac{1}{2}$	$6\frac{1}{2}$	5	$3\frac{3}{4}$	$3\frac{1}{4}$
O	$9\frac{1}{4}$	$7\frac{1}{2}$	$6\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{1}{4}$
P	$5\frac{3}{4}$	$4\frac{1}{2}$	4	$3\frac{1}{2}$	$2\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{3}{4}$	$1\frac{1}{4}$
Q	2	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$

TABLES FOR 2-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. I.

($\varrho = a + b \sin 2\theta$.) DIVISION-PLATE, SCALE 96.

	1	2	3	4
A	12	...	60	...
B	11	13	59	61
C	10	14	58	62
D	9	15	57	63
E	8	16	56	64
F	7	17	55	65
G	6	18	54	66
H	5	19	53	67
J	4	20	52	68
K	3	21	51	69
L	2	22	50	70
M	1	23	49	71
N	96	24	48	72
O	95	25	47	73
P	94	26	46	74
Q	93	27	45	75
R	92	28	44	76
S	91	29	43	77
T	90	30	42	78
U	89	31	41	79
V	88	32	40	80
W	87	33	39	81
X	86	34	38	82
Y	85	35	37	83
Z	...	36	...	84

TABLES FOR 2-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. II.

Differences of Eccentricity.

	Values of Δe, when							
b =	100	80	75	60	50	40	30	25
A	1	¾	¾	½	½	½	¼	¼
B	2½	2	1¾	1½	1¼	1	¾	½
C	4¼	3¼	3	2½	2	1½	1¼	1
D	5¾	4½	4¼	3¼	2¾	2¼	1½	1¼
E	7¼	6	5¾	4½	3¾	3	2¼	2
F	8½	6¾	6¼	5	4¼	3½	2½	2
G	10	8	7½	6	5	4	3	2¼
H	11	8¾	8¼	6½	5½	4½	3¼	2½
J	11½	9½	8¾	7	5¾	4½	3½	2¾
K	12½	10	9¼	7½	6¼	5	3¾	3
L	13	10¼	9¾	7¾	6½	5	4	3¼
M	13	10½	9¾	8	6½	5¼	4	3¼
N	13	10½	9¾	8	6½	5¼	4	3¼
O	13	10¼	9¾	7¾	6½	5	4	3¼
P	12½	10	9¼	7½	6¼	5	3¾	3
Q	11½	9½	8¾	7	5¾	4½	3½	2¾
R	11	8¾	8¼	6½	5½	4½	3¼	2½
S	10	8	7½	6	5	4	3	2¼
T	8½	6¾	6¼	5	4¼	3½	2½	2
U	7¼	6	5¾	4½	3¾	3	2¼	2
V	5¾	4½	4¼	3¼	2¾	2¼	1½	1¼
W	4¼	3¼	3	2½	2	1½	1¼	1
X	2½	2	1¾	1½	1¼	1	¾	½
Y	1	¾	¾	½	½	½	¼	¼

TABLES FOR ⅓-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. I.

($e = a + b \sin \frac{2}{3}\theta$.) Division-Plate, Scale 96.

	1	2	3	4	5	6
A	16	...	80	...	48	...
B	15	17	79	81	47	49
C	14	18	78	82	46	50
D	13	19	77	83	45	51
E	12	20	76	84	44	52
F	11	21	75	85	43	53
G	10	22	74	86	42	54
H	9	23	73	87	41	55
J	8	24	72	88	40	56
K	7	25	71	89	39	57
L	6	26	70	90	38	58
M	5	27	69	91	37	59
N	4	28	68	92	36	60
O	3	29	67	93	35	61
P	2	30	66	94	34	62
Q	1	31	65	95	33	63
R	96	32	64	96	32	64
S	95	33	63	1	31	65
T	94	34	62	2	30	66
U	93	35	61	3	29	67
V	92	36	60	4	28	68
W	91	37	59	5	27	69
X	90	38	58	6	26	70
Y	89	39	57	7	25	71
Z	88	40	56	8	24	72
A^2	87	41	55	9	23	73
B^2	86	42	54	10	22	74
C^2	85	43	53	11	21	75
D^2	84	44	52	12	20	76
E^2	83	45	51	13	19	77
F^2	82	46	50	14	18	78
G^2	81	47	49	15	17	79
H^2	...	48	...	16	...	80

TABLES FOR $\frac{3}{2}$-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. II.

DIFFERENCES OF ECCENTRICITY.

	Values of Δe, when							
b =	100	80	75	60	50	40	30	25
A	½	½	¼	¼	¼	¼	0	0
B	1½	1¼	1	¾	¾	½	¼	¼
C	2½	2	1¾	1½	1¼	1	¾	½
D	3¼	2½	2¼	2	1½	1¼	1	¾
E	4¼	3½	3	2½	2	1¾	1¼	1
F	5	4	3¾	3	2½	2	1½	1¼
G	5¾	4½	4½	3½	3	2¼	1¾	1½
H	6½	5¼	4¾	4	3¼	2½	2	1½
J	7¼	5¾	5	4½	3½	2¾	2¼	1¾
K	7¾	6¼	6	4¾	4	3	2½	2
L	8½	6¾	6¼	5	4¼	3¼	2½	2
M	8¾	7	6¾	5¼	4½	3½	2¾	2¼
N	9¼	7¼	6¾	5½	4½	3¾	2¾	2¼
O	9½	7½	7¼	5¾	4¾	3¾	3	2½
P	9¾	7¾	7½	5¾	5	4	3	2½
Q	9¾	7¾	7½	6	5	4	3	2½
R	9¾	7¾	7½	6	5	4	3	2½
S	9¾	7¾	7½	5¾	5	4	3	2½
T	9½	7½	7¼	5¾	4¾	3¾	3	2½
U	9¼	7¼	6¾	5½	4½	3¾	2¾	2¼
V	8¾	7	6¾	5¼	4½	3½	2¾	2¼
W	8½	6¾	6¼	5	4¼	3¼	2½	2
X	7¾	6¼	6	4¾	4	3	2½	2
Y	7¼	5¾	5	4½	3½	2¾	2¼	1¾
Z	6½	5¼	4¾	4	3¼	2½	2	1½
A^2	5¾	4½	4½	3½	3	2¼	1¾	1½
B^2	5	4	3¾	3	2½	2	1½	1¼
C^2	4¼	3½	3	2½	2	1¾	1¼	1
D^2	3¼	2½	2¼	2	1½	1¼	1	¾
E^2	2½	2	1¾	1½	1¼	1	¾	½
F^2	1½	1¼	1	¾	¾	½	¼	¼
G^2	½	½	¼	¼	¼	¼	0	0

TABLES FOR $\frac{5}{2}$-LOOPED FIGURES AND THEIR DERIVATIVES.—No. I.

($e = a + b \sin \frac{5}{2}\theta$.) DIVISION-PLATE, SCALE 120.

	1	2	3	4	5	6	7	8	9	10
A	12	...	60	...	108	...	36	...	84	...
B	11	13	59	61	107	109	35	37	83	85
C	10	14	58	62	106	110	34	38	82	86
D	9	15	57	63	105	111	33	39	81	87
E	8	16	56	64	104	112	32	40	80	88
F	7	17	55	65	103	113	31	41	79	89
G	6	18	54	66	102	114	30	42	78	90
H	5	19	53	67	101	115	29	43	77	91
J	4	20	52	68	100	116	28	44	76	92
K	3	21	51	69	99	117	27	45	75	93
L	2	22	50	70	98	118	26	46	74	94
M	1	23	49	71	97	119	25	47	73	95
N	120	24	48	72	96	120	24	48	72	96
O	119	25	47	73	95	1	23	49	71	97
P	118	26	46	74	94	2	22	50	70	98
Q	117	27	45	75	93	3	21	51	69	99
R	116	28	44	76	92	4	20	52	68	100
S	115	29	43	77	91	5	19	53	67	101
T	114	30	42	78	90	6	18	54	66	102
U	113	31	41	79	89	7	17	55	65	103
V	112	32	40	80	88	8	16	56	64	104
W	111	33	39	81	87	9	15	57	63	105
X	110	34	38	82	86	10	14	58	62	106
Y	109	35	37	83	85	11	13	59	61	107
Z	...	36	...	84	...	12	...	60	...	108

TABLE FOR $\frac{5}{2}$-LOOPED FIGURES AND THEIR DERIVATIVES.—N°. II.

DIFFERENCES OF ECCENTRICITY.

Values of Δe, when								
b =	100	80	75	60	50	40	30	25
A	1	¾	¾	¾	½	½	¼	¼
B	2½	2	2	1½	1¼	1	¾	½
C	4¼	3½	3¼	2½	2	1¾	1¼	1
D	5¾	4½	4¼	3½	3	2¼	1¾	1½
E	7¼	5¾	5½	4½	3½	2¾	2¼	1¾
F	8¾	7	6½	5¼	4½	3½	2½	2¼
G	9¾	7¾	7¼	6	5	4	3	2½
H	11	8¾	8¼	6½	5½	4½	3¼	2¾
J	11¾	9½	8¾	7	5¾	4¾	3½	3
K	12¼	9¾	9¼	7½	6	5	3¾	3
L	12¾	10¼	9½	7¾	6¼	5¼	4	3¼
M	13	10½	9¾	8	6½	5¼	4	3¼
N	13	10½	9¾	8	6½	5¼	4	3¼
O	12¾	10¼	9½	7¾	6¼	5¼	4	3¼
P	12¼	9¾	9¼	7½	6	5	3¾	3
Q	11¾	9½	8¾	7	5¾	4¾	3½	3
R	11	8¾	8¼	6½	5½	4½	3¼	2¾
S	9¾	7¾	7¼	6	5	4	3	2½
T	8¾	7	6½	5¼	4½	3½	2½	2¼
U	7¼	5¾	5½	4½	3½	2¾	2¼	1¾
V	5¾	4½	4¼	3½	3	2½	1¾	1½
W	4¼	3½	3¼	2½	2	1¾	1¼	1
X	2½	2	2	1½	1¼	1	¾	½
Y	1	¾	¾	¾	½	½	½	¼

TABLES FOR CIRCULAR FIGURES.—No. I.

Division-Plate, Scale 96.

	I.		III.		V.		VII.	
	1	2	1	2	1	2	1	2
A	96	48	24	72	48	96	72	24
B	1	47	25	71	49	95	73	23
C	2	46	26	70	50	94	74	22
D	3	45	27	69	51	93	75	21
E	4	44	28	68	52	92	76	20
F	5	43	29	67	53	91	77	19
G	6	42	30	66	54	90	78	18
H	7	41	31	65	55	89	79	17
J	8	40	32	64	56	88	80	16
K	9	39	33	63	57	87	81	15
L	10	38	34	62	58	86	82	14
M	11	37	35	61	59	85	83	13
N	12	36	36	60	60	84	84	12
O	13	35	37	59	61	83	85	11
P	14	34	38	58	62	82	86	10
Q	15	33	39	57	63	81	87	9
R	16	32	40	56	64	80	88	8
S	17	31	41	55	65	79	89	7
T	18	30	42	54	66	78	90	6
U	19	29	43	53	67	77	91	5
V	20	28	44	52	68	76	92	4
W	21	27	45	51	69	75	93	3
X	22	26	46	50	70	74	94	2
Y	23	25	47	49	71	73	95	1
Z	24	24	48	48	72	72	96	96

TABLES FOR CIRCULAR FIGURES.—N°. II.

Division-Plate, Scale 96.

	II.		IV.		VI.		VIII.	
	1	2	1	2	1	2	1	2
A	12	60	36	84	60	12	84	36
B	13	59	37	83	61	11	85	35
C	14	58	38	82	62	10	86	34
D	15	57	39	81	63	9	87	33
E	16	56	40	80	64	8	88	32
F	17	55	41	79	65	7	89	31
G	18	54	42	78	66	6	90	30
H	19	53	43	77	67	5	91	29
J	20	52	44	76	68	4	92	28
K	21	51	45	75	69	3	93	27
L	22	50	46	74	70	2	94	26
M	23	49	47	73	71	1	95	25
N	24	48	48	72	72	96	96	24
O	25	47	49	71	73	95	1	23
P	26	46	50	70	74	94	2	22
Q	27	45	51	69	75	93	3	21
R	28	44	52	68	76	92	4	20
S	29	43	53	67	77	91	5	19
T	30	42	54	66	78	90	6	18
U	31	41	55	65	79	89	7	17
V	32	40	56	64	80	88	8	16
W	33	39	57	63	81	87	9	15
X	34	38	58	62	82	86	10	14
Y	35	37	59	61	83	85	11	13
Z	36	36	60	60	84	84	12	12

TABLES FOR CIRCULAR FIGURES.—N°. III.

Division-Plate, Scale 96.

	I.		II.		III.		IV.		V.		VI.	
	1	2	1	2	1	2	1	2	1	2	1	2
A	96	48	16	64	32	80	48	96	64	16	80	32
B	1	47	17	63	33	79	49	95	65	15	81	31
C	2	46	18	62	34	78	50	94	66	14	82	30
D	3	45	19	61	35	77	51	93	67	13	83	29
E	4	44	20	60	36	76	52	92	68	12	84	28
F	5	43	21	59	37	75	53	91	69	11	85	27
G	6	42	22	58	38	74	54	90	70	10	86	26
H	7	41	23	57	39	73	55	89	71	9	87	25
J	8	40	24	56	40	72	56	88	72	8	88	24
K	9	39	25	55	41	71	57	87	73	7	89	23
L	10	38	26	54	42	70	58	86	74	6	90	22
M	11	37	27	53	43	69	59	85	75	5	91	21
N	12	36	28	52	44	68	60	84	76	4	92	20
O	13	35	29	51	45	67	61	83	77	3	93	19
P	14	34	30	50	46	66	62	82	78	2	94	18
Q	15	33	31	49	47	65	63	81	79	1	95	17
R	16	32	32	48	48	64	64	80	80	96	96	16
S	17	31	33	47	49	63	65	79	81	95	1	15
T	18	30	34	46	50	62	66	78	82	94	2	14
U	19	29	35	45	51	61	67	77	83	93	3	13
V	20	28	36	44	52	60	68	76	84	92	4	12
W	21	27	37	43	53	59	69	75	85	91	5	11
X	22	26	38	42	54	58	70	74	86	90	6	10
Y	23	25	39	41	55	57	71	73	87	89	7	9
Z	24	24	40	40	56	56	72	72	88	88	8	8

TABLES FOR CIRCULAR FIGURES.—No. IV.

Eccentricities.

Values of *e*, when								
d =	100	80	75	60	50	40	30	25
A	0	0	0	0	0	0	0	0
B	6½	5¼	5	4	3¼	2½	2	1¾
C	13	10½	9¾	8	6½	5	4	3¼
D	19½	15½	14½	11¾	9¾	7¾	5¾	4¾
E	26	20¾	19½	15½	13	10¼	7¾	6½
F	32	25½	24	19¼	16	12¾	9½	8
G	38¼	30½	28½	23	19	15¼	11½	9½
H	44¼	35½	33	26½	22	17¾	13¼	11
J	50	40	37½	30	25	20	15	12½
K	55½	44½	41¼	33¼	27¾	22¼	16½	13¾
L	61	48¾	45¾	36½	30½	24½	18¼	15¼
M	66	52¾	49½	39½	33	26½	19¾	16½
N	70¾	56½	53	42½	35¼	28¼	21¼	17¾
O	75¼	60¼	56½	45	37¾	30	22½	18¾
P	79½	63½	59½	47½	39¾	31¾	23¾	19¾
Q	83	66½	62¼	49¾	41½	33¼	25	20¾
R	86½	69¼	64¾	52	43¼	34½	26	21¾
S	89¾	71¾	67¼	53¾	44¾	35¾	27	22½
T	92½	74	69¼	55½	46¼	37	27¾	23
U	94¾	75¾	71	56¾	47¼	37¾	28½	23¾
V	96½	77¼	72¼	58	48¼	38¼	29	24
W	98	78½	73½	58¾	49	39¼	29½	24½
X	99	79¼	74¼	59½	49½	39½	29¾	24¾
Y	99¾	79¾	74½	59¾	49¾	39¾	29¾	24¾
Z	100	80	75	60	50	40	30	25

TABLE FOR STRAIGHT LINES.—No. I.

Division-Plate, Scale 96.

p =	100	80	60	50	40	30	25	20
A	100	80	60	50	40	30	25	20
B	100¼	80¼	60	50	40	30	25	20
C	100¾	80¾	60½	50½	40¼	30¼	25¼	20
D	102	81½	61¼	51	40¾	30½	25½	20¼
E	103½	82¾	62	51¾	41¼	31	25¾	20¾
F	105½	84½	63¼	52¾	42¼	31¾	26¼	21
G	108¼	86½	65	54	43¼	32½	27¼	21¾
H	111½	89¼	67	55¾	44¾	33½	27¾	22¼
J	115½	92½	69¼	57¾	46¼	34½	28¾	23
K	120¼	96¼	72¼	60	48	36	30	24
L	126	100¾	75½	63	50¼	37¾	31½	25
M	133	106½	79¾	66½	53¼	38¾	33¼	26¾
N	141½	113¼	85	70¾	56½	42½	35¼	28¼
O	151¾	121½	91	75¾	60¾	45½	37¾	30¼
P	164¼	131½	98½	82	65¾	49¼	41	32¾
Q	180	144	108	90	72	54	45	36
R	200	160	120	100	80	60	50	40

TABLE FOR STRAIGHT LINES.—N°. II.

DIVISION-PLATE, SCALE 120.

$p =$	100	80	75	60	50	40	25
A	100	80	75	60	50	40	25
B	100	80	75	60	50	40	25
C	100½	80½	75¼	60¼	50¼	40¼	25
D	101¼	81	76	60¾	50½	40½	25¼
E	102¼	81¾	76¾	61¼	51	41	25½
F	103½	82¾	77½	62	51¾	41½	25¾
G	105	84	78¾	63	52½	42	26¼
H	107	85½	80¼	64¼	53½	42¾	26¾
J	109½	87½	82	65¾	54¾	43¾	27¼
K	112¼	89¾	84¼	67½	56	45	28
L	115½	92¼	86½	69¼	57¾	46½	28¾
M	119¼	96¼	89¼	71¼	59½	47¾	29¾
N	123½	98¾	92½	73	61¾	49½	30¾
O	128¾	103	96¼	77	64¼	51½	32
P	134½	107½	101	80¾	67¼	53¾	33¾
Q	141½	113½	106	85	70¾	56½	35¼
R	149½	121½	112	87	74¾	59¾	37¼
S	159	127¼	119¼	95½	79½	63½	39¾
T	170	136	127½	102	85	68	42½
U	183½	146¾	137½	110	91¾	73¼	45¾
V	200	160	150	120	100	80	50

TABLE FOR ELLIPSES.—No. I.

a =	100	90	80	70	60	50	40
A	100	90	80	70	60	50	40
B	99¾	89¾	80	70	60	50	40
C	99½	89½	79½	69½	59¾	49¾	39¾
D	99	89	79¼	69¼	59½	49½	39¾
E	98¼	88¼	78½	68¾	59	49	39¼
F	97¼	87½	77¾	68	58¼	48½	38¾
G	96	86½	76¾	67¼	57¾	48	38½
H	95	85½	76	66½	57¾	47½	38
J	93½	84¼	75	65½	56¼	46¾	37½
K	92¼	83	73¾	64½	55½	46¼	37
L	91	81¾	72¾	63¾	54½	45½	36½
M	89¾	80¾	71½	62¾	53¾	44¾	35¾
N	88¼	79½	70¾	61¾	53	44¼	35½
O	87	78¼	69¾	61	52¼	43½	34¾
P	86	77¼	68¾	60¼	51½	42¾	34¼
Q	84¾	76¼	68	59½	51	42½	34
R	83¾	75½	67	58¾	50¼	42	33½
S	83	74¾	66¼	58	49¾	41½	33¼
T	82¼	74	65¾	57½	49¼	41	37¾
U	81½	73¼	65¼	57	49	40¾	32½
V	81	72¾	64¾	56½	48½	40½	32½
W	80½	72½	64½	56½	48¼	40¼	32¼
X	80¼	72¼	64¼	56¼	48¼	40	32
Y	80	72	64	56	48	40	32
Z	80	72	64	56	48	40	32

TABLE FOR ELLIPSES.—N°. II.

$a =$	100	90	80	70	60	50	40
A	100	90	80	70	60	50	40
B	99¾	98¾	79¾	69¾	60	50	40
C	99¼	89¼	79¼	69½	59½	49½	39¾
D	98¼	88½	78¾	68¾	59	49	39¼
E	97	87¼	77½	67¾	58¼	48½	38¾
F	95½	85¾	76¼	66¾	57¼	47¾	38¼
G	93½	84¼	75	65½	56¼	46¾	37½
H	91¾	82½	73½	64¼	55	45¾	36¾
J	89¾	80¾	71¾	62¾	53¾	45	36
K	87¾	79	70¼	61½	52¾	44	35
L	85¾	77¼	68¾	60	51½	43	34¼
M	84	75½	61¼	58¾	50½	42	33½
N	82¼	74	65¾	57½	49¼	41	32¾
O	80½	72½	64½	56¼	48¼	40¼	32¼
P	79	71	63¼	55¼	47¼	39½	31½
Q	77½	69¾	62	54¼	46½	38¾	31
R	76¼	68¾	61	53¼	45¾	38	30½
S	75	67½	60	52½	45	37½	30
T	74	66¾	59¼	51¾	44½	37	29¾
U	73¼	66	58¾	51¼	44	36¾	29¼
V	72½	65¼	58	50¾	43½	36¼	29
W	72	64¾	57¾	50½	43¼	36	28¾
X	71½	64½	57¼	50	43	35¾	28¾
Y	71½	64¼	57¼	50	43	35¾	28½
Z	71½	64¼	57¼	50	42¾	35¾	28½

TABLE FOR ELLIPSES.—N°. III.

Division-Plate, Scale 96.

a =	100	90	80	70	60	50	40
A	100	90	80	70	60	50	40
B	99½	89½	79½	69¾	59¾	49¾	39¾
C	98½	88¾	78¾	69	59	49¼	39½
D	96¾	87	77½	67¾	58	48½	39
E	94½	85	75½	66¼	56¾	47¼	37¾
F	92	82¾	73½	64½	55½	46	36¾
G	89	80	71¼	62¼	53½	44½	35½
H	86¼	77½	69	60½	51¾	43	34½
J	83¼	75	66½	58¼	50	41½	33¾
K	80¼	72½	64¼	56¼	48¼	40	32
L	77½	69¾	62	54¼	46½	38¾	31
M	75	67¼	60	52½	45	37½	30
N	72¾	65½	58½	50	43¾	36½	29
O	70½	63½	56½	49½	42¼	35¼	28
P	68¾	62	55	48	41	34¼	27½
Q	67	60¼	53½	47	40¼	33½	26¾
R	65½	58	52½	46	39¼	32¾	26½
S	64½	57¾	51½	45¼	38¾	32¼	25¾
T	63	56¾	50½	44	37¾	31½	25½
U	62	55¾	49½	43½	37¼	31	24¾
V	61¼	55	49	43	36¾	30¾	24½
W	60¾	54¾	48½	42½	36½	30½	24¼
X	60¼	54¼	48¼	42¼	36¼	30¼	24
Y	60	54	48	42	36	30	24
Z	60	54	48	42	36	30	24

TABLE FOR ELLIPSES.—N°. IV.

a =	100	90	80	70	60	50	40
A	100	90	80	70	60	50	40
B	99	89¼	79¼	69¼	59	49½	39½
C	96½	87	77¼	67¾	57¾	48¼	38½
D	93	83¾	74¼	65	55¾	46½	37¼
E	88	79¼	70½	61¾	52¾	44	35¼
F	83¼	75	66½	58¼	50	41¾	33¼
G	78½	70¾	62¾	54¾	47	39¼	31½
H	73¾	66½	59	51½	44¼	37	29½
J	69½	62½	55½	48¾	41¾	33¾	27¾
K	65¾	59¼	52½	46	40½	33	26¼
L	62¼	56	49¾	43½	37¼	31	25
M	59¼	53¼	47¼	41½	35½	29½	23¾
N	56½	51	45¼	39½	34	28¼	22½
O	54	48½	43¼	37¾	32½	27	21½
P	52	46¾	41¾	36½	31¼	26	20¾
Q	50¼	45¼	40¼	35¼	30¼	25¼	20
R	48¾	44	39	34¼	29¼	24½	19½
S	47½	42¾	38	33¼	28½	23¾	19
T	46½	42	37¼	32½	27¾	23¼	18½
U	45½	41	36½	32	27¼	22¾	18¼
V	44¾	40¼	36	31¼	26¾	22½	18
W	44¼	39¾	35¾	31	26½	22¼	17¾
X	44	39½	35¼	30¾	26½	22	17½
Y	43¾	39¼	35	30½	26¼	21¾	17½
Z	43½	39¼	34¾	30½	26¼	21¾	17½

TABLE FOR WAVED ELLIPSES.

	I.	II.	III.	IV.
A	100	100	100	100
B	99¼	97¾	99¼	97¾
C	97¾	96¼	97½	96
D	95¼	95¼	94½	94½
E	96½	95	95¼	93¾
F	96¾	95¼	95	93½
G	96	96	93½	93½
H	94½	93	91¼	89¾
J	91¾	90¼	88	86½
K	88½	88½	84	84
L	89¼	87⅓	84	82½
M	89¼	87¾	83½	82
N	88½	88½	82¼	82¼
O	86½	85	80	78½
P	84¼	82¾	77¼	75¾
Q	81	81	77¾	73¾
R	82	80½	74½	73
S	82½	81	74½	73
T	82¼	82¼	74	74
U	81	79½	72¾	71¼
V	79¼	77¾	70¾	69¼
W	76¾	76¾	68¼	68¼
X	78½	77	69¾	68¼
Y	79½	78	71	69½
Z	80	80	71½	71½

TABLE OF THE TRIGONOMETRICAL FUNCTIONS OF THE ANGLES CORRESPONDING TO THE NUMBERS ON THE DIVISION-PLATE, SCALE 96.

	Sine	Covers	Cosec	Tang	Cotang	Secant	Versin	Cosine	
96	0	1	∞	0	∞	1·000	0	1·000	24
1	·065	·935	15·290	·066	15·257	1·002	·002	·998	23
2	·131	·869	7·661	·132	7·596	1·009	·009	·991	22
3	·195	·805	5·126	·199	5·027	1·020	·019	·981	21
4	·259	·741	3·864	·268	3·732	1·035	·034	·966	20
5	·321	·679	3·111	·339	2·946	1·056	·053	·947	19
6	·383	·617	2·613	·414	2·414	1·082	·076	·924	18
7	·442	·558	2·261	·493	2·028	1·115	·103	·897	17
8	·500	·500	2·000	·577	1·732	1·155	·134	·866	16
9	·556	·444	1·800	·668	1·497	1·203	·169	·831	15
10	·609	·391	1·643	·767	1·303	1·260	·207	·793	14
11	·659	·341	1·517	·877	1·140	1·330	·248	·752	13
12	·707	·293	1·414	1·000	1·000	1·414	·293	·707	12
	Cosine	Versin	Secant	Cotang	Tang	Cosec	Covers	Sine	

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THE END.

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